Package ‘sparseEigen’

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Title Computation of Sparse Eigenvectors of a Matrix

Version 0.1.0

Description Computation of sparse eigenvectors of a matrix (aka sparse PCA) with running time 2-3 orders of magnitude lower than existing methods and better final performance in terms of recovery of sparsity pattern and estimation of numerical values. Can handle covariance matrices as well as data matrices with real or complex-valued entries. Different levels of sparsity can be specified for each individual ordered eigenvector and the method is robust in parameter selection. See vignette for a detailed documentation and comparison, with several illustrative examples. The package is based on the paper: K. Benidis, Y. Sun, P. Babu, and D. P. Palomar (2016). "Orthogonal Sparse PCA and Covariance Estimation via Procrustes Reformulation," IEEE Transactions on Signal Processing <doi:10.1109/TSP.2016.2605073>.

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URL https://github.com/dppalomar/sparseEigen,
https://www.danielppalomar.com,
https://doi.org/10.1109/TSP.2016.2605073

BugReports https://github.com/dppalomar/sparseEigen/issues

Depends R (>= 3.4.0)

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Encoding UTF-8

LazyData true

Imports

RoxygenNote 6.0.1

Suggests knitr, MASS, rmarkdown, testthat

VignetteBuilder knitr

NeedsCompilation no

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spEigen

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spEigen  Sparse Spectral Decomposition of a Matrix

Description

Computes sparse (orthogonal) eigenvectors of covariance matrix or directly of data matrix.

Usage

spEigen(X, q = 1, rho = 0.5, data = FALSE, d = NA, V = NA,
   thres = 1e-09)

Arguments

X  m-by-m covariance matrix or n-by-m data matrix (n samples, m variables). Both
   real and complex matrices are accepted.
q  number of eigenvectors to be estimated.
rho  sparsity weight factor. Any nonnegative number (suggested range [0,1]).
data  boolean variable. If TRUE, X is treated as a data matrix, else as a covariance
   matrix (default).
d  vector with q weights. The default value is seq(from = 1, to = 0.5, length.out
   = q).
V  initial m-by-q matrix point. If not provided, the eigenvectors of the sample
   covariance matrix are used.
thres  threshold value. All the entries of the sparse eigenvectors less or equal to thres
   are set to 0. The default value is 1e-9.

Value

A list with the following components:

vectors  m-by-q matrix, columns corresponding to the q leading sparse eigenvectors.
standard_vectors  m-by-q matrix, columns corresponding to standard (non-sparse) leading eigenvectors.
values  vector with the q leading eigenvalues (in decreasing order).
spEigenCov

Author(s)
Konstantinos Benidis and Daniel P. Palomar

References

Examples

```r
library(sparseEigen)

n <- 100  # samples
m <- 500  # dimension
q <- 3    # number of sparse eigenvectors to be estimated
sp_card <- 0.1*m  # sparsity of each eigenvector

# generate covariance matrix with sparse eigenvectors
V <- matrix(0, m, q)
V[cbind(seq(1, q*sp_card), rep(1:q, each = sp_card))] <- 1/sqrt(sp_card)
V <- cbind(V, matrix(rnorm(m*(m-q)), m, m-q))
V <- qr.Q(qr(V))  # orthogonalize eigenvectors
lmd <- c(100*seq(from = q, to = 1), rep(1, m-q))  # generate eigenvalues
R <- V %*% diag(lmd) %*% t(V)  # covariance matrix

# generate data
X <- MASS::mvrnorm(n, rep(0, m), R)  # random data with underlying sparse structure

# standard and sparse eigenvectors
res_standard <- eigen(cov(X))
res_sparse <- spEigen(cov(X), q)

# show inner product between estimated eigenvectors and originals (the closer to 1 the better)
abs(diag(t(res_standard$vectors) %*% V[, 1:q]))  # for standard estimated eigenvectors
abs(diag(t(res_sparse$vectors) %*% V[, 1:q]))  # for sparse estimated eigenvectors
```

Description
Estimates the covariance matrix with sparse (orthogonal) eigenvectors (in other words, it jointly estimates the sparse eigenvectors and the eigenvalues).

Usage

```r
spEigenCov(S, q = 1, rho = 0.5, thres = 1e-09)
```
Arguments

- **S**: m-by-m sample covariance matrix. It is required that S is full-rank. Both real and complex matrices are accepted.
- **q**: number of sparse eigenvectors.
- **rho**: sparsity weight factor. Any nonnegative number (suggested range [0,1]).
- **thres**: threshold value. All the entries of the sparse eigenvectors less or equal to thres are set to 0. The default value is \(1e^{-9}\).

Value

A list with the following components:

- **vectors**: m-by-m matrix, columns corresponding to eigenvectors.
- **values**: m-by-1 vector corresponding to eigenvalues.

Author(s)

Konstantinos Benidis and Daniel P. Palomar

References


Examples

```r
## Not run:
library(sparseEigen)
n <- 600 # samples
m <- 500 # dimension
q <- 3 # number of sparse eigenvectors to be estimated
sp_card <- 0.1*m # sparsity of each eigenvector

# generate covariance matrix with sparse eigenvectors
V <- matrix(0, m, q)
V[cbind(seq(1, q*sp_card), rep(1:q, each = sp_card))] <- 1/sqrt(sp_card)
V <- cbind(V, matrix(rnorm(m*(m-q)), m, m-q)) # orthogonalize eigenvectors
lmd <- c(100*seq(from = q, to = 1), rep(1, m-q)) # generate eigenvalues
R <- V %*% diag(lmd) %*% t(V) # covariance matrix

# generate data
X <- MASS::mvrnorm(n, rep(0, m), R) # random data with underlying sparse structure

# standard and sparse estimation
res_standard <- eigen(cov(X))
res_sparse <- spEigenCov(cov(X), q)

# show inner product between estimated eigenvectors and originals (the closer to 1 the better)
```
abs(diag(t(res_standard$vectors) %*% V[, 1:q]))  # for standard estimated eigenvectors
abs(diag(t(res_sparse$vectors) %*% V[, 1:q]))  # for sparse estimated eigenvectors

# show error between estimated and true covariance
norm(cov(X) - R, type = 'F')  # for sample covariance matrix
norm(res_sparse$cov - R, type = 'F')  # for covariance with sparse eigenvectors

## End(Not run)
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