Package ‘spc’

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Title Statistical Process Control -- Calculation of ARL and Other
    Control Chart Performance Measures
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Depends R (>= 1.8.0)
Description Evaluation of control charts by means of
    the zero-state, steady-state ARL (Average Run Length) and RL quantiles.
    Setting up control charts for given in-control ARL. The control charts
    under consideration are one- and two-sided EWMA, CUSUM, and
    Shiryaev-Roberts schemes for monitoring the mean or variance of normally
    distributed independent data. ARL calculation of the same set of schemes un-
    der drift (in the mean) are added.
    Eventually, all ARL measures for the multivariate EWMA (MEWMA) are provided.
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**dphat**

Percent defective for normal samples

---

**Description**

Density, distribution function and quantile function for the sample percent defective calculated on normal samples with mean equal to \( \mu \) and standard deviation equal to \( \sigma \).

**Usage**

```
dphat(x, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)
pphat(q, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)
qphat(p, n, mu=0, sigma=1, type="known", LSL=-3, USL=3, nodes=30)
```

**Arguments**

- \( x, q \) vector of quantiles.
- \( p \) vector of probabilities.
- \( n \) sample size.
- \( mu, sigma \) parameters of the underlying normal distribution.
- \( type \) choose whether the standard deviation is given and fixed ("known") or estimated and potentially monitored ("estimated").
- \( LSL, USL \) lower and upper specification limit, respectively.
- \( nodes \) number of quadrature nodes needed for \( type="estimated" \).

**Details**

Bruhn-Suhr/Krumbholz (1990) derived the cumulative distribution function of the sample percent defective calculated on normal samples to applying them for a new variables sampling plan. These results were heavily used in Krumbholz/Zoller (1995) for Shewhart and in Knoth/Steinmetz (2013) for EWMA control charts. For algorithmic details see, essentially, Bruhn-Suhr/Krumbholz (1990). Two design variants are treated: The simple case, \( type="known" \), with known normal variance and the presumably much more relevant and considerably intricate case, \( type="estimated" \), where both parameters of the normal distribution are unknown. Basically, given lower and upper specification limits and the normal distribution, one estimates the expected yield based on a normal sample of size \( n \).
Value

Returns vector of pdf, cdf or qf values for the statistic phat.

Author(s)

Sven Knoth

References


See Also

`phat.ewma.arl` for routines using the herewith considered phat statistic.

Examples

```r
# Figures 1 (c) and (d) from Knoth/Steinmetz (2013)

n <- 5
LSL <- -3
USL <- 3

par(mar=c(5, 5, 1, 1) + 0.1)

p.star <- 2*pnorm((LSL-USL)/2)  # for p <= p.star pdf and cdf vanish

p_ <- seq(p.star+1e-10, 0.07, 0.0001)  # define support of Figure 1

# Figure 1 (c)

pp_ <- pphat(p_, n)
plot(p_, pp_, type="l", xlab="p", ylab=expression(P( hat(p) <= p )),
     xlim=c(0, 0.06), ylim=c(0,1), lwd=2)
abline(h=0:1, v=p.star, col="grey")

# Figure 1 (d)

dp_ <- dphat(p_, n)
plot(p_, dp_, type="l", xlab="p", ylab="f(p)", xlim=c(0, 0.06),
     ylim=c(0,50), lwd=2)
abline(h=0, v=p.star, col="grey")
```
Description

Computation of the (zero-state) Average Run Length (ARL) at given Poisson mean \(\mu\).

Usage

\[
euklid.ewma.arl(gX, gY, kL, kU, \mu, y0, r0=0)
\]

Arguments

- \(gX\): first and
- \(gY\): second integer forming the rational \(\lambda = \frac{gX}{gX+gY}\), \(\lambda\) mimics the usual EWMA smoothing constant.
- \(kL\): lower control limit of the NCS-EWMA control chart, integer.
- \(kU\): upper control limit of the NCS-EWMA control chart, integer.
- \(\mu\): mean value of Poisson distribution.
- \(y0\): headstart like value – it is proposed to use the in-control mean.
- \(r0\): further element of the headstart – deviating from the default should be done only in case of full understanding of the scheme.

Details

A new idea of applying EWMA smoothing to count data based on integer division with remainders. It is highly recommended to read the corresponding paper (see below).

Value

Return single value which resemble the ARL.

Author(s)

Sven Knoth

References


See Also

later.
Examples

# RCM (2015), Table 12, page 243, first NCS column

\[ g_X = 5 \]
\[ g_Y = 24 \]
\[ k_L = 16 \]
\[ k_U = 24 \]
\[ \mu_0 = 20 \]

\[ L_0 = \text{euklid.ewma.arl}(g_X, g_Y, k_L, k_U, \mu_0, \mu_0) \]

# should be 1219.2

---

**lns2ewma.arl**

*Compute ARLs of EWMA ln \( S^2 \) control charts (variance charts)*

### Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts (based on the log of the sample variance \( S^2 \)) monitoring normal variance.

### Usage

\[ \text{lns2ewma.arl}(l, c_l, c_u, \sigma, df, hs=NULL, \text{sided}="upper", r=40) \]

### Arguments

- **l**: smoothing parameter \( \lambda \) of the EWMA control chart.
- **c_l**: lower control limit of the EWMA control chart.
- **c_u**: upper control limit of the EWMA control chart.
- **\sigma**: true standard deviation.
- **df**: actual degrees of freedom, corresponds to subsample size (for known mean it is equal to the subsample size, for unknown mean it is equal to subsample size minus one).
- **hs**: so-called headstart (enables fast initial response) – the default value (hs=NULL) corresponds to the in-control mean of \( \ln S^2 \).
- **\text{sided}**: distinguishes between one- and two-sided two-sided \( \text{EWMA-S}^2 \) control charts by choosing "upper" (upper chart with reflection at \( c_l \)), "lower" (lower chart with reflection at \( c_u \)), and "two" (two-sided chart), respectively.
- **r**: dimension of the resulting linear equation system: the larger the better.

### Details

\( \text{lns2ewma.arl} \) determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.
Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts for monitoring normal mean.

Examples

```r
lns2ewma.arl <- Vectorize("lns2ewma.arl", "sigma")

## Crowder/Hamilton (1992)
## moments of ln S^2
E_log_gamma <- function(df) log(2/df) + digamma(df/2)
V_log_gamma <- function(df) trigamma(df/2)
E_log_gamma_approx <- function(df) -1/df - 1/3/df^2 + 2/15/df^4
V_log_gamma_approx <- function(df) 2/df + 2/df^2 + 4/3/df^3 - 16/15/df^5

## results from Table 3 ( upper chart with reflection at 0 = log(sigma0=1) )
## original entries are (lambda = 0.05, K = 1.06, df=n-1=4)
# sigma ARL
# 1 200
# 1.1 43
# 1.2 18
# 1.3 11
# 1.4 7.6
# 1.5 6.0
# 2 3.2
def <- 4
lambda <- .05
K <- 1.06
cu <- K * sqrt( lambda/(2-lambda) * V_log_gamma_approx(df) )
sigmas <- c(1 + (0:5)/10, 2)
arls <- round(lns2ewma.arl(lambda, 0, cu, sigmas, df, hs=0, sided="upper"), digits=1)
data.frame(sigmas, arls)

## Knoth (2005)
## compare with Table 3 (p. 351)
```
lambda <- .05
df <- 4
K <- 1.05521
cu <- 1.05521 * sqrt( lambda/(2-lambda) * V_log_gamma_approx(df) )

## upper chart with reflection at sigma0=1 in Table 4
## original entries are
# sigma ARL_0 ARL_-0.267
# 1 200.0 200.0
# 1.1 43.04 41.55
# 1.2 18.10 19.92
# 1.3 10.75 13.11
# 1.4 7.63 9.93
# 1.5 5.97 8.11
# 2 3.17 4.67

M <- -0.267
cuM <- lns2ewma.crit(lambda, 200, df, cl=M, hs=M, r=60)[2]
arls1 <- round(lns2ewma.ARL(lambda, 0, cu, sigmas, df, hs=0, sided="upper"), digits=2)
arls2 <- round(lns2ewma.ARL(lambda, M, cuM, sigmas, df, hs=M, sided="upper", r=60), digits=2)
data.frame(sigmas, arls1, arls2)

lns2ewma.crit

Compute critical values of EWMA ln S^2 control charts (variance charts)

Description

Computation of the critical values (similar to alarm limits) for different types of EWMA control charts (based on the log of the sample variance S^2) monitoring normal variance.

Usage

lns2ewma.crit(l, L0, df, sigma0=1, cl=NULL, cu=NULL, hs=NULL, sided="upper", mode="fixed", r=40)

Arguments

1 smoothing parameter lambda of the EWMA control chart.
L0 in-control ARL.
df actual degrees of freedom, corresponds to subsample size (for known mean it is equal to the subsample size, for unknown mean it is equal to subsample size minus one.
sigma0 in-control standard deviation.
cl deployed for sided="upper", that is, upper variance control chart with lower reflecting barrier cl.
cu for two-sided (sided="two") and fixed upper control limit (mode="fixed"), for all other cases cu is ignored.
so-called headstart (enables fast initial response) – the default value (hs=NULL) corresponds to the in-control mean of $\ln S^2$.

sided distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart with reflection at cl), "lower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.

mode only deployed for sided="two" – with "fixed" an upper control limit (see cu) is set and only the lower is calculated to obtain the in-control ARL $L_0$, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated). With "vanilla" limits symmetric around the in-control mean of $\ln S^2$ are determined, while for "eq.tails" the in-control ARL values of two single EWMA variance charts (decompose the two-sided scheme into one lower and one upper scheme) are matched.

$r$ dimension of the resulting linear equation system: the larger the more accurate.

Details

$lns2ewma.crit$ determines the critical values (similar to alarm limits) for given in-control ARL $L_0$ by applying secant rule and using $lns2ewma.arl()$. In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at $\sigma_0$. See Knoth (2010) and the related example.

Value

Returns the lower and upper control limit cl and cu.

Author(s)

Sven Knoth

References


See Also

$lns2ewma.arl$ for calculation of ARL of EWMA $\ln S^2$ control charts.
Examples

```r
## Knoth (2005)
## compare with 1.05521 mentioned on page 350 third line from below
L0 <- 200
lambda <- .05
df <- 4
limits <- lns2ewma.crit(lambda, L0, df, cl=0, hs=0)
limits["cu"]/sqrt( lambda/(2-lambda)*(2/df+2/df^2+4/3/df^3-16/15/df^5) )
```

---

**mewma.arl**  
Compute ARLs of MEWMA control charts

**Description**

Computation of the (zero-state) Average Run Length (ARL) for multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

**Usage**

```r
mewma.arl(l, cE, p, delta=0, hs=0, r=20, ntype=NULL, qm0=20, qm1=qm0)
mewma.arl.f(l, cE, p, delta=0, r=20, ntype=NULL, qm0=20, qm1=qm0)
mewma.ad(l, cE, p, delta=0, r=20, n=20, type="cond", hs=0, ntype=NULL, qm0=20, qm1=qm0)
```

**Arguments**

- `l`: smoothing parameter lambda of the MEWMA control chart.
- `cE`: alarm threshold of the MEWMA control chart.
- `p`: dimension of multivariate normal distribution.
- `delta`: magnitude of the potential change, `delta=0` refers to the in-control state.
- `hs`: so-called headstart (enables fast initial response) – must be non-negative.
- `r`: number of quadrature nodes – dimension of the resulting linear equation system for `delta = 0`. For non-zero `delta` this dimension is mostly `r^2` (Markov chain approximation leads to some larger values). Caution: If `ntype` is set to "co" (collocation), then values of `r` larger than 20 lead to large computing times. For the other selections this would happen for values larger than 40.
- `ntype`: choose the numerical algorithm to solve the ARL integral equation. For `delta=0`: Possible values are "gl", "gl2" (gauss-legendre, classic and with variables change: square), "co" (collocation, for `delta > 0` with sin transformation), "ra" (radau), "cc" (clenshaw-curtis), "mc" (markov chain), and "sr" (simpson rule). For `delta` larger than 0, some more values besides the others are possible: "gl3", "gl4", "gl5" (gauss-legendre with a further change in variables: sin, tan, sinh), "co2", "co3" (collocation with some trimming and tan as quadrature stabilizing transformations, respectively). If it is set to NULL (the
mewma.arl

default), then for delta=0 then "gl2" is chosen. If delta larger than 0, then for p equal 2 or 4 "gl3" and for all other values "gl5" is taken. "ra" denotes the method used in Rigdon (1995a). "mc" denotes the Markov chain approximation.

type
switch between "cond" and "cyc1" for differentiating between the conditional (no false alarm) and the cyclical (after false alarm re-start in hs), respectively.
n
number of quadrature nodes for Calculating the steady-state ARL integral(s).

qm0, qm1
number of collocation quadrature nodes for the out-of-control case (qm0 for the inner integral, qm1 for the outer one), that is, for positive delta, and for the in-control case (now only qm0 is deployed) if via ntype the collocation procedure is requested.

Details

Basically, this is the implementation of different numerical algorithms for solving the integral equation for the MEWMA in-control (delta = 0) ARL introduced in Rigdon (1995a) and out-of-control (delta != 0) ARL in Rigdon (1995b). Most of them are nothing else than the Nystroem approach – the integral is replaced by a suitable quadrature. Here, the Gauss-Legendre (more powerful), Radau (used by Rigdon, 1995a), Clenshaw-Curtis, and Simpson rule (which is really bad) are provided. Additionally, the collocation approach is offered as well, because it is much better for small odd values for p. FORTRAN code for the Radau quadrature based Nystroem of Rigdon (1995a) was published in Bodden and Rigdon (1999) – see also http://lib.stat.cmu.edu/jqt/31-1. Furthermore, FORTRAN code for the Markov chain approximation (in- and out-of-control) could be found at http://lib.stat.cmu.edu/jqt/33-4. The related papers are Runger and Prabhu (1996) and Molnau et al. (2001). The idea of the Clenshaw-Curtis quadrature was taken from Capizzi and Masarotto (2010), who successfully deployed a modified Clenshaw-Curtis quadrature to calculate the ARL of combined (univariate) Shewhart-EWMA charts. It turns out that it works also nicely for the MEWMA ARL. The version mewma.arl.f() without the argument hs provides the ARL as function of one (in-control) or two (out-of-control) arguments.

Value

Returns a single value which is simply the zero-state ARL.

Author(s)

Sven Knoth

References

Kevin M. Bodden and Steven E. Rigdon (1999), A program for approximating the in-control ARL for the MEWMA chart, Journal of Quality Technology 31(1), 120-123.


mewma.arl


See Also

mewma.crit for getting the alarm threshold to attain a certain in-control ARL.

Examples

```r
# Rigdon (1995a), p. 357, Tab. 1
p <- 2
r <- 0.25
h4 <- c(8.37, 9.90, 11.89, 13.36, 14.82, 16.72)
for (i in 1:length(h4)) cat(paste(h4[i], "\t", round(mewma.arl(r, h4[i], p, ntype="ra"), 2), "\n"))

r <- 0.1
h4 <- c(6.98, 8.63, 10.77, 12.37, 13.88, 15.88)
for (i in 1:length(h4)) cat(paste(h4[i], "\t", round(mewma.arl(r, h4[i], p, ntype="ra"), 2), "\n"))

# Rigdon (1995b), p. 372, Tab. 1
## Not run:
r <- 0.1
p <- 4
h <- 12.73
for (sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3))
  cat(paste(sdelta, "\t",
           round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), 2), "\n"))

p <- 5
h <- 14.56
for (sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3))
  cat(paste(sdelta, "\t",
           round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), 2), "\n"))

p <- 10
h <- 22.67
for (sdelta in c(0, 0.125, 0.25, .5, 1, 2, 3))
  cat(paste(sdelta, "\t",
           round(mewma.arl(r, h, p, delta=sdelta^2, ntype="ra", r=25), 2), "\n"))
```
## End(Not run)

# Runger/Prabhu (1996), p. 1704, Tab. 1
## Not run:

```r
r <- 0.1
p <- 4
H <- 12.73

cat(paste(0, "\t", round(mewma.arl(r, H, p, delta=0, ntype="mc", r=50), digits=2), "\n"))
for ( delta in c(.5, 1.5, 2, 3) )
  cat(paste(delta, "\t",
         round(mewma.arl(r, H, p, delta=delta, ntype="mc", r=25), digits=2), "\n"))
```

# compare with Fortran program (MEWMA-ARLs.f90) from Molnau et al. (2001) with m1 = m2 = 25

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```
p <- 20
H <- 37.01

cat(paste(0, "\t",
         round(mewma.arl(r, H, p, delta=0, ntype="mc", r=50), digits=2), "\n"))
for ( delta in c(.5, 1.5, 2, 3) )
  cat(paste(delta, "\t",
         round(mewma.arl(r, H, p, delta=delta, ntype="mc", r=25), digits=2), "\n"))
```

# compare with Fortran program (MEWMA-ARLs.f90) from Molnau et al. (2001) with m1 = m2 = 25

<table>
<thead>
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<td>37.01</td>
<td>20.00</td>
<td>0.10</td>
<td>0.00</td>
<td>199.09</td>
</tr>
<tr>
<td>37.01</td>
<td>20.00</td>
<td>0.50</td>
<td>61.62</td>
<td></td>
</tr>
<tr>
<td>37.01</td>
<td>20.10</td>
<td>1.00</td>
<td>20.17</td>
<td></td>
</tr>
<tr>
<td>37.01</td>
<td>20.15</td>
<td>1.50</td>
<td>11.40</td>
<td></td>
</tr>
<tr>
<td>37.01</td>
<td>20.20</td>
<td>2.00</td>
<td>8.03</td>
<td></td>
</tr>
<tr>
<td>37.01</td>
<td>20.30</td>
<td>3.00</td>
<td>5.18</td>
<td></td>
</tr>
</tbody>
</table>

## End(Not run)

# Knoth (2017), p. 85, Tab. 3, rows with p=3
## Not run:

```r
p <- 3
lambda <- 0.05
h4 <- mewma.crit(lambda, 200, p)
benchmark <- mewma.arl(lambda, h4, p, delta=1, r=50)
```

```r
mc.arl <- mewma.arl(lambda, h4, p, delta=1, r=25, ntype="mc")
ra.arl <- mewma.arl(lambda, h4, p, delta=1, r=27, ntype="ra")
c0.arl <- mewma.arl(lambda, h4, p, delta=1, r=12, ntype="co2")
g13.arl <- mewma.arl(lambda, h4, p, delta=1, r=30, ntype="gl3")
g15.arl <- mewma.arl(lambda, h4, p, delta=1, r=25, ntype="gl5")
```

abs( benchmark - data.frame(mc.arl, ra.arl, co.arl, g13.arl, g15.arl) )
## End(Not run)

# Prabhu/Runger (1997), p. 13, Tab. 3
## Not run:
\[
p \leftarrow 2\\
r \leftarrow 0.1\\
H \leftarrow 8.64
\]
cat(paste0(0, "\t",
        round(mewma.ad(r, H, p, delta=0, type="cycl", ntype="mc", r=60), digits=2), "\n"))
for ( delta in c(0.5, 1, 1.5, 2, 3) )
cat(paste(delta, "\t",
        round(mewma.ad(r, H, p, delta=delta, type="cycl", ntype="mc", r=30), digits=2), "\n"))

# better accuracy
for ( delta in c(0, 0.5, 1, 1.5, 2, 3) )
cat(paste(delta, "\t",
        round(mewma.ad(r, H, p, delta=delta^2, type="cycl", ntype="mc", r=30), digits=2), "\n"))

## End(Not run)

---

### mewma.crit

**Compute alarm threshold of MEWMA control charts**

#### Description

Computation of the alarm threshold for multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

#### Usage

```r
mewma.crit(\(l\), \(L0\), \(p\), \(hs=0\), \(r=20\))
```

#### Arguments

- `\(l\)`: smoothing parameter lambda of the MEWMA control chart.
- `\(L0\)`: in-control ARL.
- `\(p\)`: dimension of multivariate normal distribution.
- `\(hs\)`: so-called headstart (enables fast initial response) – must be non-negative.
- `\(r\)`: number of quadrature nodes – dimension of the resulting linear equation system.

#### Details

`mewma.crit` determines the alarm threshold of for given in-control ARL `\(L0\)` by applying secant rule and using `mewma.arl()` with `ntype="gl2"`

#### Value

Returns a single value which resembles the critical value `\(c\)`.
mewma.psi

**Author(s)**
Sven Knoth

**References**

**See Also**
mewma.arl for zero-state ARL computation.

**Examples**

```r
# Rigdon (1995), p. 358, Tab. 1
p <- 4
L0 <- 500
r <- .25
h4 <- mewma.crit(r, L0, p)
h4
## original value is 16.38.

# Knoth (2017), p. 82, Tab. 2
p <- 3
L0 <- 1e3
lambda <- c(0.25, 0.2, 0.15, 0.1, 0.05)
h4 <- rep(NA, length(lambda))
for (i in 1:length(lambda)) h4[i] <- mewma.crit(lambda[i], L0, p, r=20)
round(h4, digits=2)
## original values are
## 15.82 15.62 15.31 14.76 13.60
```

---

**mewma.psi**

*Compute steady-state density of the MEWMA statistic*

**Description**

Computation of the (zero-state) steady-state density function of the statistic deployed in multivariate exponentially weighted moving average (MEWMA) charts monitoring multivariate normal mean.

**Usage**

```r
mewma.psi(l, cE, p, type="cond", hs=0, r=20)
```
Arguments

- **l**: smoothing parameter lambda of the MEWMA control chart.
- **cE**: alarm threshold of the MEWMA control chart.
- **p**: dimension of multivariate normal distribution.
- **type**: switch between "cond" and "cyc1" for differentiating between the conditional (no false alarm) and the cyclical (after false alarm re-start in hs), respectively.
- **hs**: the re-starting point for the cyclical steady-state framework.
- **r**: number of quadrature nodes.

Details

Basically, ideas from Knoth (2017, MEWMA numerics) and Knoth (2016, steady-state ARL concepts) are merged. More details will follow.

Value

Returns a function.

Author(s)

Sven Knoth

References


See Also

mewma.arl for calculating the in-control ARL of MEWMA.

Examples

```r
lambda <- 0.1
L0 <- 200
p <- 3
h4 <- mewma.crit(lambda, L0, p)
x_ <- seq(0, h4*lambda/(2-lambda), by=0.002)
psi <- mewma.psi(lambda, h4, p)
psi_ <- psi(x_)
# plot(x_, psi_, type="l", xlab="x", ylab=expression(psi(x)), xlim=c(0,1.2))
# cf. to Figure 1 in Knoth (2018), p. 514, p=3
```
**Description**

Computation of the (zero-state) Average Run Length (ARL) at given rate $p$.

**Usage**

```r
p.ewma.arl(lambda, ucl, n, p, z0, sided="upper", lcl=NULL, d.res=1, r.mode="ieee.round", i.mode="integer")
```

**Arguments**

- `lambda` smoothing parameter of the EWMA $p$ control chart.
- `ucl` upper control limit of the EWMA $p$ control chart.
- `n` subgroup size.
- `p` (failure/success) rate.
- `z0` so-called headstart (give fast initial response).
- `sided` distinguishes between one- and two-sided EWMA control chart by choosing "upper", "lower", and "two", respectively.
- `lcl` lower control limit of the EWMA $p$ control chart; needed for two-sided design.
- `d.res` resolution (see details).
- `r.mode` round mode – allowed modes are "gan.floor", "floor", "ceil", "ieee.round", "round", "mix".
- `i.mode` type of interval center – "integer" or "half" integer.

**Details**

The monitored data follow a binomial distribution with size $n$ and failure/success probability $p$. The ARL values of the resulting EWMA control chart are determined by Markov chain approximation. Here, the original EWMA values are approximated by multiples of one over $d.res$. Different ways of rounding (see `r.mode`) to the next multiple are implemented. Besides Gan's paper nothing is published about the numerical subtleties.

**Value**

Return single value which resemble the ARL.

**Author(s)**

Sven Knoth
References


See Also

later.

Examples

```r
## Gan (1990)

# Table 1

n <- 150
p0 <- .1
z0 <- n*p0

lambda <- c(1, .51, .165)
u <- c(27, 22, 18)

p.value <- .1 + (0:20)/200

p.EWMA.arl <- Vectorize(p.ewma.arl, "p")

arl1.value <- round(p.EWMA.arl(lambda[1], hu[1], n, p.value, z0, r.mode="round"), digits=2)
arl2.value <- round(p.EWMA.arl(lambda[2], hu[2], n, p.value, z0, r.mode="round"), digits=2)
arl3.value <- round(p.EWMA.arl(lambda[3], hu[3], n, p.value, z0, r.mode="round"), digits=2)

arls <- matrix(c(arl1.value, arl2.value, arl3.value), ncol=length(lambda))
rownames(arls) <- p.value
colnames(arls) <- paste("lambda =", lambda)
arls

## Knoth/Steinmetz (2013)

n <- 5
p0 <- 0.02
z0 <- n*p0

lambda <- 0.3
ucl <- 0.649169922 ## in-control ARL 370.4 (determined with d.res = 2^14 = 16384)

res.list <- 2^(1:11)
arl.list <- NULL
for ( res in res.list ) {
  arl <- p.ewma.arl(lambda, ucl, n, p0, z0, d.res=res)
  arl.list <- c(arl.list, arl)
}
cbind(res.list, arl.list)
```
Compute ARLs of EWMA phat control charts

Description

Computation of the (zero-state) Average Run Length (ARL), upper control limit (ucl) for given in-control ARL, and lambda for minimal out-of control ARL at given shift.

Usage

phat.ewma.arl(lambda, ucl, mu, n, z0, sigma=1, type="known", LSL=-3, USL=3, N=15, qm=25, ntype="coll")

phat.ewma.crit(lambda, L0, mu, n, z0, sigma=1, type="known", LSL=-3, USL=3, N=15, qm=25)

phat.ewma.lambda(L0, mu, n, z0, sigma=1, type="known", max_l=1, min_l=.001, LSL=-3, USL=3, qm=25)

Arguments

lambda smoothing parameter of the EWMA control chart.
ucl upper control limit of the EWMA phat control chart.
L0 pre-defined in-control ARL (Average Run Length).
mu true mean or mean where the ARL should be minimized (then the in-control mean is simply 0).
n subgroup size.
z0 so-called headstart (gives fast initial response).
type choose whether the standard deviation is given and fixed ("known") or estimated and potentially monitored ("estimated").
sigma actual standard deviation of the data – the in-control value is 1.
max_l, min_l maximal and minimal value for optimal lambda search.
LSL,USL lower and upper specification limit, respectively.
N size of collocation base, dimension of the resulting linear equation system is equal to N.
qm number of nodes for collocation quadratures.
ntype switch between the default method coll (collocation) and the classic one markov (Markov chain approximation) for calculating the ARL numerically.

Details

The three implemented functions allow to apply a new type control chart. Basically, lower and upper specification limits are given. The monitoring vehicle then is the empirical probability that an item will not follow these specification given the sequence of sample means. If the related EWMA sequence violates the control limits, then the alarm indicates a significant process deterioration. For details see the paper mentioned in the references. To be able to construct the control charts, see the first example.
Value

Return single values which resemble the ARL, the critical value, and the optimal lambda, respectively.

Author(s)

Sven Knoth

References


See Also

`sewma.arl` for a further collocation based ARL calculation routine.

Examples

```r
## Simple example to demonstrate the chart.

# some functions
h.mu <- function(mu) pnorm(LSL-mu) + pnorm(mu-USL)
ewma <- function(x, lambda=0.1, z0=0) filter(lambda*x, 1-lambda, m="r", init=z0)

# parameters
LSL <- -3 # lower specification limit
USL <- 3 # upper specification limit
n <- 5 # batch size
lambda <- 0.1 # EWMA smoothing parameter
L0 <- 1000 # in-control Average Run Length (ARL)
z0 <- h.mu(0) # start at minimal defect level
ucl <- phat.ewma.crit(lambda, L0, 0, n, z0, LSL=LSL, USL=USL)

# data
x0 <- matrix(rnorm(50*n), ncol=5) # in-control data
x1 <- matrix(rnorm(50*n, mean=0.5), ncol=5)# out-of-control data
x <- rbind(x0,x1) # all data

# create chart
xbar <- apply(x, 1, mean)
phat <- h.mu(xbar)
z <- ewma(phat, lambda=lambda, z0=z0)
plot(1:length(z), z, type="l", xlab="batch", ylim=c(0,.02))
abline(h=z0, col="grey", lwd=.7)
abline(h=ucl, col="red")
```

## S. Knoth, S. Steinmetz (2013)

# Table 1
pois.cusum.arl

Compute ARLs of Poisson CUSUM control charts

Description
Computation of the (zero-state) Average Run Length (ARL) at given mean \( \mu \).

Usage
```r
pois.cusum.arl(mu, km, hm, m, i0=0, sided="upper", rando=FALSE,
               gamma=0, km2=0, hm2=0, m2=0, i02=0, gamma2=0)
```

Arguments
- \( \mu \) actual mean.
- \( km \) enumerator of rational approximation of reference value \( k \).
hm

denominator of rational approximation of reference value.

m

eenumerator of rational approximation of reference value.

i0

head start value as integer multiple of 1/m; should be an element of 0:hm.

sided
distinguishes between different one- and two-sided CUSUM control chart by choosing "upper", "lower" and "two", respectively.

rando

Switch for activating randomization in order to allow continuous ARL control.

gamma

Randomization probability. If the CUSUM statistic is equal to the threshold h, an control chart alarm is triggered with probability gamma.

km2,hm2,m2,i02,gamma2

corresponding values of the second CUSUM chart (to building a two-sided CUSUM scheme).

Details

The monitored data follow a Poisson distribution with mu. The ARL values of the resulting EWMA control chart are determined via Markov chain calculations. We follow the algorithm given in Lucas (1985) expanded with some arithmetic 'tricks' (e.g., by deploying Toeplitz matrix algebra).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

later.
Examples

```r
k <- .25
h <- 10
m <- 4
km <- m * k
hm <- m * h
mu0 <- 1 * k
ARL <- pois.cusum.arl(mu0, km, hm-1, m)
# Lucas reported 438 (in Table 2, first block, row 10.0 .25 .0 ..., column 1.0
# Recall that Lucas and other trigger an alarm, if the CUSUM statistic is greater than
# or equal to the alarm threshold h
print(ARL)

ARL <- pois.cusum.arl(mu0, km, hm-1, m, i0=round((hm-1)/2))
# Lucas reported 333 (in Table 3, first block, row 10.0 .25 .0 ..., column 1.0
print(ARL)

## Lucas 1985, lower chart (Tables 4 and 5)
ARL <- pois.cusum.arl(mu0, km, hm-1, m, sided="lower")
# Lucas reported 437 (in Table 4, first block, row 10.0 .25 .0 ..., column 1.0
print(ARL)

ARL <- pois.cusum.arl(mu0, km, hm-1, m, i0=round((hm-1)/2), sided="lower")
# Lucas reported 318 (in Table 5, first block, row 10.0 .25 .0 ..., column 1.0
print(ARL)
```

pois.cusum.crit

Compute alarm thresholds and randomization constants of Poisson CUSUM control charts

Description

Computation of the CUSUM upper limit and, if needed, of the randomization probability, given mean mu0.

Usage

```r
pois.cusum.crit(mu0, km, A, m, i0=0, sided="upper", rando=FALSE)
```

Arguments

- `mu0` actual in-control mean.
- `km` enumerator of rational approximation of reference value k.
- `A` target in-control ARL (average run length).
- `m` denominator of rational approximation of reference value.
- `i0` head start value as integer multiple of 1/m; should be an element of 0:100 (a more reasonable upper limit will be established soon).
sided
distinguishes between different one- and two-sided CUSUM control chart by choosing "upper", "lower" and "two", respectively.

rando
Switch for activating randomization in order to allow continuous ARL control.

Details
The monitored data follow a Poisson distribution with $\mu$ (here the in-control level $\mu_0$). The ARL values of the resulting EWMA control chart are determined via Markov chain calculations. With some grid search, we obtain the smallest value for the integer threshold component $h_m$ so that the resulting ARL is not smaller than $A$. If equality is needed, then activating $rando=TRUE$ yields the corresponding randomization probability $\gamma$. More details will follow in a paper that will be submitted in 2020.

Value
Returns two single values, integer threshold $h_m$ resulting in the final alarm threshold $h=h_m/m$, and the randomization probability.

Author(s)
Sven Knoth

References

See Also
later.

Examples
```r
## Lucas 1985
mu0 <- 0.25
km <- 1
A <- 430
m <- 4
```
```r
#cv <- pois.cusum.crit(mu0, km, A, m)
cv <- c(40, 0)
# Lucas reported h = 10 alias hm = 40 (in Table 2, first block, row 10.0 .25 .0 ..., column 1.0
# Recall that Lucas and other trigger an alarm, if the CUSUM statistic is greater than
# or equal to the alarm threshold h
print(cv)
```

---

**pois.cusum.crit.L0L1**  
*Compute the CUSUM k and h for given in-control ARL L0 and out-of-control ARL L1, Poisson case*

---

**Description**

Computation of the reference value k and the alarm threshold h for one-sided CUSUM control charts monitoring Poisson data, if the in-control ARL L0 and the out-of-control ARL L1 are given.

**Usage**

```r
pois.cusum.crit.L0L1(mu0, L0, L1, sided="upper", OUTPUT=FALSE)
```

**Arguments**

- `mu0` in-control Poisson mean.
- `L0` in-control ARL.
- `L1` out-of-control ARL.
- `sided` distinguishes between "upper" and "lower" CUSUM designs.
- `OUTPUT` controls whether iteration details are printed.

**Details**

`pois.cusum.crit.L0L1` determines the reference value k and the alarm threshold h for given in-control ARL L0 and out-of-control ARL L1 by applying grid search and using `pois.cusum.arl()` and `pois.cusum.crit()`. These CUSUM design rules were firstly (and quite rarely afterwards) used by Ewan and Kemp. In the Poisson case, Rossi et al. applied them while analyzing three different normal approximations of the Poisson distribution. See the example which illustrates the validity of all these approaches.

**Value**

Returns a data frame with results for the denominator m of the rational approximation, km as (integer) numerator of the reference value approximation, the resulting out-of-control mean mu1, the final approximation k of the reference value, the threshold values hm (integer) and h (=hm/m), and the randomization constant gamma (the target in-control ARL is exactly matched).

**Author(s)**

Sven Knoth
References


See Also

`pois.cusum.arl` for zero-state ARL and `pois.cusum.crit` for threshold h computation.

Examples

```r
## Table 1 from Rossi et al. (1999) -- one-sided CUSUM
La <- 500 # in-control ARL
Lr <- 7 # out-of-control ARL
m_a <- 0.52 # in-control mean of the Poisson variate
## Not run: kh <- xcusum.crit.L0L1(La, Lr, sided="one") # instead of deploying EK1960, one could use more accurate numbers
EK_k <- 0.60 # EK1960 results in
EK_h <- 3.80 # Table 2 on p. 372
eZR <- 2*EK_h # reproduce normal ooc mean from reference value k
m_r <- 1.58 # EK1960 Table 3 on p. 377 for m_a = 0.52
R1 <- round( eZR/sqrt(m_a) + 1, digits=2)
R2 <- round(( eZR/2/sqrt(m_a) + 1 )^2, digits=2)
R3 <- round(( sqrt(4 + 2*eZR/sqrt(m_a)) - 1 )^2, digits=2)
RS <- round( m_r / m_a, digits=2 )
## Not run: K_hk <- pois.cusum.crit.L0L1(m_a, La, Lr) # 'Var' 'Var' 'Var'
K_hk <- data.frame(m=1000, km=948, mu1=1.563777, k=0.948, hm=3832, h=3.832, gamma=0.1201901)
# get k for competing means mu0 (m_a) and mu1 (m_r)
k_m01 <- function(mu0, mu1) (mu1 - mu0) / (log(mu1) - log(mu0))
# get ooc mean mu1 (m_r) for given mu0 (m_a) and reference value k
m1_km0 <- function(mu0, k) {
    zero <- function(x) k - k_m01(mu0,x)
    upper <- mu0 + .5
    while ( zero(upper) > 0 ) upper <- upper + 0.5
    mu1 <- uniroot(zero, c(mu0*1.00000001, upper), tol=1e-9)$root
    mu1
}
K_m_r <- m1_km0(m_a, K_hk$k)
RK <- round( K_m_r / m_a, digits=2 )
cat(paste(m_a, R1, R2, R3, RS, RK, "\n", sep="\t"))
```
Usage

pois.ewma.arl(lambda, AL, AU, mu0, z0, mu, sided="two", rando=FALSE, gL=0, gU=0, mcdesign="transfer", N=101)

Arguments

lambda smoothing parameter of the EWMA p control chart.
AL, AU factors to build the lower and upper control limit, respectively, of the Poisson EWMA control chart.
mu0 in-control mean.
z0 so-called headstart (give fast initial response).
mu actual mean.
sided distinguishes between one- and two-sided EWMA control chart by choosing "upper", "lower", and "two", and "zwei", respectively.
rando Switch between the standard limit treatment, FALSE, and an additional randomisation (to allow 'perfect' ARL calibration) by setting TRUE. If randomisation is used, then set the corresponding probabilities, gL and gU, appropriately.
gL, gU If the EWMA statistic is at the limit (approximately), then an alarm is triggered with probability gL and gU for the lower and upper limit, respectively.
mcdesign choose either "classic" which follows Borror, Champ and Rigdon (1998), or the more sophisticated "transfer" which improves the accuracy heavily.
N number of states of the approximating Markov chain; is equal to the dimension of the resulting linear equation system.

Details

The monitored data follow a Poisson distribution with mu. The ARL values of the resulting EWMA control chart are determined by Markov chain approximation. We follow the algorithm given in Borror, Champ and Rigdon (1998).

Value

Return single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

later.
Examples

```r
## Borror, Champ and Rigdon (1998), Table 2, PEWMA column
mu0 <- 20
lambda <- 0.27
A <- 3.319
mu1 <- c(2*(3:15), 35)
ARL1 <- rep(NA, length(mu1))
for (i in 1:length(mu1))
  ARL1[i] <- pois.ewma.arl(lambda, A, A, mu0, mu0, mu1[i], mcdesign="classic")
print(cbind(mu1, round(ARL1, digits=1)))
```

**pois.ewma.crit**  
*Compute ARLs of Poisson EWMA control charts*

Description

Computation of the (zero-state) Average Run Length (ARL) at given mean \( \mu \).

Usage

```r
pois.ewma.crit(lambda, L0, mu0, z0, AU=3, sided="two", design="sym", rando=FALSE, mcdesign="transfer", N=101, jmax=4)
```

Arguments

- `lambda`  
  smoothing parameter of the EWMA p control chart.
- `L0`  
  value of the so-called in-control Average Run Length (ARL) for the Poisson EWMA control chart.
- `mu0`  
  in-control mean.
- `z0`  
  so-called headstart (give fast initial response).
- `AU`  
  in case of the lower chart deployed as reflecting upper barrier – might be increased step by step until the resulting lower limit does not change anymore.
- `sided`  
  distinguishes between one- and two-sided EWMA control chart by choosing "upper", "lower", and "two", respectively.
- `design`  
  distinguishes between limits symmetric to the in-control mean \( \mu_0 \) and an ARL-unbiased design (ARL maximum at \( \mu_0 \)).
- `rando`  
  Switch between the standard limit treatment, FALSE, and an additional randomisation (to allow 'perfect' ARL calibration) by setting TRUE. If randomisation is used, then the corresponding probabilities, \( g_L \) and \( g_U \) are determined, appropriately.
- `mcdesign`  
  choose either "classic" which follows Borror, Champ and Rigdon (1998), or the more sophisticated "transfer" which improves the accuracy heavily.
- `N`  
  number of states of the approximating Markov chain; is equal to the dimension of the resulting linear equation system.
- `jmax`  
  number of digits for the to be calculated factors \( A \) (sort of accuracy).
Details

The monitored data follow a Poisson distribution with $\mu$. Here we solve the inverse task to the usual ARL calculation. Hence, determine the control limit factors so that the in-control ARL is (roughly) equal to $L_0$. The ARL values underneath the routine are determined by Markov chain approximation. The algorithm is just a grid search that takes care of the discrete ARL behavior.

Value

Return one or two values being the control limit factors.

Author(s)

Sven Knoth

References


See Also

later.

Examples

```r
## Borror, Champ and Rigdon (1998), page 30, original value is $A = 2.8275$
mu0 <- 4
lambda <- 0.2
L0 <- 351
A <- pois.ewma.crit(lambda, L0, mu0, mu0, mcdesign="classic")
print(round(A, digits=4))
```

---

`quadrature.nodes.weights`

*Calculate quadrature nodes and weights*

Description

Computation of the nodes and weights to enable numerical quadrature.

Usage

`quadrature.nodes.weights(n, type="GL", x1=-1, x2=1)`
Arguments

- **n**: number of nodes (and weights).
- **type**: quadrature type – currently Gauss-Legendre, "GL", and Radau, "Ra", are supported.
- **x1**: lower limit of the integration interval.
- **x2**: upper limit of the integration interval.

Details

A more detailed description will follow soon. The algorithm for the Gauss-Legendre quadrature was delivered by Knut Petras to me, while the one for the Radau quadrature was taken from John Burkardt.

Value

Returns two vectors which hold the needed quadrature nodes and weights.

Author(s)

Sven Knoth

References


John Burkardt (2015), [https://people.sc.fsu.edu/~jburkardt/f_src/quadrule/quadrule.html](https://people.sc.fsu.edu/~jburkardt/f_src/quadrule/quadrule.html)

See Also

Many of the ARL routines use the Gauss-Legendre nodes.

Examples

```r
# GL
n <- 10
qnw <- quadrature.nodes.weights(n, type="GL")
qnw

# Radau
n <- 10
qnw <- quadrature.nodes.weights(n, type="Ra")
qnw
```
Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts (based on the sample variance $S^2$) monitoring normal variance.

Usage

```
scusum.arl(k, h, sigma, df, hs=0, sided="upper", k2=NULL, h2=NULL, hs2=0, r=40, qm=30, version=2)
```

Arguments

- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **sigma**: true standard deviation.
- **df**: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided two-sided CUSUM-$S^2$ control charts by choosing "upper" (upper chart), "lower" (lower chart), and "two" (two-sided chart), respectively. Note that for the two-sided chart the parameters "k2" and "h2" have to be set too.
- **k2**: In case of a two-sided CUSUM chart for variance the reference value of the lower chart.
- **h2**: In case of a two-sided CUSUM chart for variance the decision interval of the lower chart.
- **hs2**: In case of a two-sided CUSUM chart for variance the headstart of the lower chart.
- **r**: Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals – see Knoth 2006).
- **qm**: Number of quadrature nodes for calculating the collocation definite integrals.
- **version**: Distinguish version numbers (1,2,...). For internal use only.

Details

`scusum.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (piecewise Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.
Author(s)

Sven Knoth

References


See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts for monitoring normal mean.

Examples

```r
## Knoth (2006)
## compare with Table 1 (p. 507)
k <- 1.46 # sigma1 = 1.5
df <- 1
h <- 10

# original values
# sigma coll63 BE Hawkins MC 10^9 (s.e.)
# 1  260.7369 260.7546 261.32 260.7399 (0.0081)
# 1.1 90.1319 90.1389 90.31 90.1319 (0.0027)
# 1.2 43.6867 43.6897 43.75 43.6845 (0.0013)
# 1.3 26.2916 26.2932 26.32 26.2929 (0.0007)
# 1.4 18.1231 18.1239 18.14 18.1235 (0.0005)
# 1.5 13.6268 13.6273 13.64 13.6272 (0.0003)
# 2 5.9904 5.9910 5.99 5.9903 (0.0001)

# replicate the column coll63
sigma <- c(1, 1.1, 1.2, 1.3, 1.4, 1.5, 2)
arl <- rep(NA, length(sigma))
for (i in 1:length(sigma))
  arl[i] <- round(scusum.arl(k, h, sigma[i], df, r=63, qm=20, version=2), digits=4)
data.frame(sigma, arl)
```

scusum.crit

*Compute decision intervals of CUSUM control charts (variance charts)*

Description

Computation of the decision intervals (alarm limits) for different types of CUSUM control charts (based on the sample variance $S^2$) monitoring normal variance.
**Usage**

```r
scusum.crit(k, L0, sigma, df, hs=0, sided="upper", mode="eq.tails",
               k2=NULL, hs2=0, r=40, qm=30)
```

**Arguments**

- **k**
  - reference value of the CUSUM control chart.
- **L0**
  - in-control ARL.
- **sigma**
  - true standard deviation.
- **df**
  - actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- **hs**
  - so-called headstart (enables fast initial response).
- **sided**
  - distinguishes between one- and two-sided two-sided CUSUM-S\(^2\) control charts by choosing "upper" (upper chart), "lower" (lower chart), and "two" (two-sided chart), respectively. Note that for the two-sided chart the parameters "k2" and "h2" have to be set too.
- **mode**
  - only deployed for sided="two" – with "eq.tails" two one-sided CUSUM charts (lower and upper) with the same in-control ARL are coupled. With "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
- **k2**
  - in case of a two-sided CUSUM chart for variance the reference value of the lower chart.
- **hs2**
  - in case of a two-sided CUSUM chart for variance the headstart of the lower chart.
- **r**
  - Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals – see Knoth 2006).
- **qm**
  - Number of quadrature nodes for calculating the collocation definite integrals.

**Details**

The `scusum.crit` function determines the decision interval (alarm limit) for given in-control ARL \( L_0 \) by applying secant rule and using `scusum.arl()`.

**Value**

Returns a single value which resembles the decision interval \( h \).

**Author(s)**

Sven Knoth
References


See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts monitoring normal mean.

Examples

```R
## Knoth (2006)
## compare with Table 1 (p. 507)
k <- 1.46 # sigma1 = 1.5
df <- 1
L0 <- 260.74
h <- scusum.crit(k, L0, 1, df)
h # original value is 10
```

**scusums.arl**

*Compute ARLs of CUSUM-Shewhart control charts (variance charts)*

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM-Shewhart combo control charts (based on the sample variance \(S^2\)) monitoring normal variance.

Usage

```R
scusums.arl(k, h, cS, sigma, df, hs=0, sided="upper", k2=NULL, h2=NULL, hs2=0, r=40, qm=30, version=2)
```

Arguments

- `k`: reference value of the CUSUM control chart.
- `h`: decision interval (alarm limit, threshold) of the CUSUM control chart.
- `cS`: Shewhart limit.
- `sigma`: true standard deviation.
- `df`: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
- `hs`: so-called headstart (enables fast initial response).
sided distinguishes between one- and two-sided two-sided CUSUM-$S^2$ control charts by choosing "upper" (upper chart), "lower" (lower chart), and "two" (two-sided chart), respectively. Note that for the two-sided chart the parameters "k2" and "h2" have to be set too.

k2 In case of a two-sided CUSUM chart for variance the reference value of the lower chart.

h2 In case of a two-sided CUSUM chart for variance the decision interval of the lower chart.

hs2 In case of a two-sided CUSUM chart for variance the headstart of the lower chart.

r Dimension of the resulting linear equation system (highest order of the collocation polynomials times number of intervals -- see Knoth 2006).

qm Number of quadrature nodes for calculating the collocation definite integrals.

version Distinguish version numbers (1,2,...). For internal use only.

Details

scusums.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (piecewise Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

scusum.arl for zero-state ARL computation of standalone CUSUM control charts for monitoring normal variance.

Examples

```r
## will follow
```
Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts (based on the sample variance $S^2$) monitoring normal variance.

Usage

sewma.arl(l, cl, cu, sigma, df, s2.on=TRUE, hs=NULL, sided="upper", r=40, qm=30)

Arguments

- **l**: smoothing parameter lambda of the EWMA control chart.
- **cl**: lower control limit of the EWMA control chart.
- **cu**: upper control limit of the EWMA control chart.
- **sigma**: true standard deviation.
- **df**: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- **s2.on**: distinguishes between $S^2$ and $S$ chart.
- **hs**: so-called headstart (enables fast initial response); the default (NULL) yields the expected in-control value of $S^2$ (1) and $S$ ($c_4$), respectively.
- **sided**: distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at cl – the actual value of cl is not used), "Rupper" (upper chart with reflection at cl), "Rlower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.
- **r**: dimension of the resulting linear equation system (highest order of the collocation polynomials).
- **qm**: number of quadrature nodes for calculating the collocation definite integrals.

Details

sewma.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of collocation (Chebyshev polynomials).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth
References


See Also

`xewma.arl` for zero-state ARL computation of EWMA control charts for monitoring normal mean.

Examples

```r
## Knoth (2005)
## compare with Table 1 (p. 347): 249.9997
## Monte Carlo with 10^9 replicates: 249.9892 +/- 0.008
l <- .025
df <- 1
cu <- 1 + 1.661865*sqrt(l/(2-l))*sqrt(2/df)
sewma.arl(l, 0, cu, 1, df)

## ARL values for upper and lower EWMA charts with reflecting barriers
## (reflection at in-control level sigma0 = 1)
## examples from Knoth (2006), Tables 4 and 5
Ssewma.arl <- Vectorize("sewma.arl", "sigma")
## upper chart with reflection at sigma0=1 in Table 4
## original entries are
# sigma ARL
# 1 100.0
# 1.01 85.3
# 1.02 73.4
# 1.03 63.5
# 1.04 55.4
# 1.05 48.7
# 1.1 27.9
# 1.2 12.9
# 1.3 7.86
# 1.4 5.57
# 1.5 4.30
# 2 2.11

## Not run:
l <- 0.15
df <- 4
cu <- 1 + 2.4831*sqrt(l/(2-l))*sqrt(2/df)
sigmas <- c(1 + (0:5)/100, 1 + (1:5)/10, 2)
arls <- round(Ssewma.arl(l, 1, cu, sigmas, df, sided="Rupper", r=100), digits=2)
data.frame(sigmas, arls)
## End(Not run)
```
sewma.arl.prerun

Compute ARLs of EWMA control charts (variance charts) in case of estimated parameters

Description

Computation of the (zero-state) Average Run Length (ARL) for EWMA control charts (based on the sample variance $S^2$) monitoring normal variance with estimated parameters.

Usage

sewma.arl.prerun(l, cl, cu, sigma, df1, df2, hs=1, sided="upper", r=40, qm=30, qm.sigma=30, truncate=1e-10)

Arguments

- `l`: smoothing parameter lambda of the EWMA control chart.
- `cl`: lower control limit of the EWMA control chart.
- `cu`: upper control limit of the EWMA control chart.
- `sigma`: true standard deviation.
- `df1`: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
- `df2`: degrees of freedom of the pre-run variance estimator.
- `hs`: so-called headstart (enables fast initial response).
- `sided`: distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at cl – the actual value of cl is not used), "Rupper" (upper chart with reflection at cl), "Rlower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.
Computation of the critical values (similar to alarm limits) for different types of EWMA control charts (based on the sample variance $S^2$) monitoring normal variance.

Usage

```r
sewma.crit(1,L0,df,sigma0=1,cl=NULL,cu=NULL,hs=NULL,s2.on=TRUE,
sided="upper",mode="fixed",ur=4,r=40,qm=30)
```
Arguments

- \( l \) : smoothing parameter lambda of the EWMA control chart.
- \( L_0 \) : in-control ARL.
- \( df \) : actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
- \( \sigma_0 \) : in-control standard deviation.
- \( cl \) : deployed for \( sided="\text{Rupper}" \), that is, upper variance control chart with lower reflecting barrier \( cl \).
- \( cu \) : for two-sided (\( sided="\text{two}" \)) and fixed upper control limit (\( mode="\text{fixed}" \)) a value larger than \( \sigma_0 \) has to be given, for all other cases \( cu \) is ignored.
- \( hs \) : so-called headstart (enables fast initial response); the default (\( \text{NULL} \)) yields the expected in-control value of \( S^2 \) (1) and \( S \left( \hat{c}_4 \right) \), respectively.
- \( s2.on \) : distinguishes between \( S^2 \) and \( S \) chart.
- \( sided \) : distinguishes between one- and two-sided two-sided EWMA-\( S^2 \) control charts by choosing "upper" (upper chart without reflection at \( cl \) – the actual value of \( cl \) is not used), "Rupper" (upper chart with reflection at \( cl \)), "Rlower" (lower chart with reflection at \( cu \)), and "two" (two-sided chart), respectively.
- \( mode \) : only deployed for \( sided="\text{two}" \) – with "fixed" an upper control limit (see \( cu \)) is set and only the lower is calculated to obtain the in-control ARL \( L_0 \), while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated). With "vanilla" limits symmetric around 1 (the in-control value of the variance) are determined, while for "eq.tails" the in-control ARL values of two single EWMA variance charts (decompose the two-sided scheme into one lower and one upper scheme) are matched.
- \( ur \) : truncation of lower chart for \( eq.tails \) mode.
- \( r \) : dimension of the resulting linear equation system (highest order of the collocation polynomials).
- \( qm \) : number of quadrature nodes for calculating the collocation definite integrals.

Details

\texttt{sewma.crit} determines the critical values (similar to alarm limits) for given in-control ARL \( L_0 \) by applying secant rule and using \texttt{sewma.arl()}. In case of \( sided="\text{two}" \) and \( mode="\text{unbiased}" \) a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at \( \sigma_0 \). See Knoth (2010) and the related example.

Value

Returns the lower and upper control limit \( cl \) and \( cu \).

Author(s)

Sven Knoth
References


See Also

sewma.arl for calculation of ARL of variance charts.

Examples

```r
## Mittag et al. (1998)
## compare their upper critical value 2.91 that
## leads to the upper control limit via the formula shown below
## (for the usual upper EWMA $S^2$).
## See Knoth (2006b) for a discussion of this EWMA setup and it's evaluation.

l <- 0.18
L0 <- 250
df <- 4
limits <- sewma.crit(l, L0, df)
limits["cu"]

limits.cu.mittag_et_al <- 1 + sqrt(l/(2-l))*sqrt(2/df)*2.91
limits.cu.mittag_et_al

## Knoth (2005)
## reproduce the critical value given in Figure 2 (c=1.661865) for
## upper EWMA $S^2$ with df=1

l <- 0.025
L0 <- 250
df <- 1
limits <- sewma.crit(l, L0, df)
cv.Fig2 <- (limits["cu"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Fig2

## the small difference (sixth digit after decimal point) stems from
```
## tighter criterion in the secant rule implemented in the R package.

## demo of unbiased ARL curves
## Deploy, please, not matrix dimensions smaller than 50 -- for the
## sake of accuracy, the value 80 was used.
## Additionally, this example needs between 1 and 2 minutes on a 1.6 Ghz box.

## Not run:
1 <- 0.1
L0 <- 500
df <- 4
limits <- sewma.crit(l, L0, df, sided="two", mode="unbiased", r=80)
SEWMA.arl <- Vectorize(sewma.arl, "sigma")
SEWMA.ARL <- function(sigma)
  SEWMA.arl(l, limits[1], limits[2], sigma, df, sided="two", r=80)
layout(matrix(1:2, nrow=1))
curve(SEWMA.ARL, .75, 1.25, log="y")
curve(SEWMA.ARL, .95, 1.05, log="y")
## End(Not run)
# the above stuff needs about 1 minute

## control limits for upper and lower EWMA charts with reflecting barriers
## (reflection at in-control level sigma0 = 1)
## examples from Knoth (2006a), Tables 4 and 5

## Not run:
## upper chart with reflection at sigma0=1 in Table 4: c = 2.4831
1 <- .15
L0 <- 100
df <- 4
limits <- sewma.crit(l, L0, df, cl=1, sided="Rupper", r=100)
cv.Tab4 <- (limits["cu"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Tab4

## lower chart with reflection at sigma0=1 in Table 5: c = 2.0613
1 <- 0.115
L0 <- 200
df <- 5
limits <- sewma.crit(l, L0, df, cu=1, sided="Rlower", r=100)
cv.Tab5 <- -(limits["cl"]-1)/( sqrt(l/(2-l))*sqrt(2/df) )
cv.Tab5
## End(Not run)

sewma.crit.prerun  Compute critical values of of EWMA (variance charts) control charts under pre-run uncertainty

### Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal variance.
Usage

sewma.crit.prerun(l,L0,df1,df2,sigma0=1,cl=NULL,cu=NULL,hs=1,sided="upper", mode="fixed",r=40,qm=30,qm.sigma=30,truncate=1e-10, tail_approx=TRUE,c.error=1e-10,a.error=1e-9)

Arguments

l       smoothing parameter lambda of the EWMA control chart.
L0      in-control quantile value.
df1     actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
df2     degrees of freedom of the pre-run variance estimator.
sigma, sigma0   true and in-control standard deviation, respectively.
cl     deployed for sided="Rupper", that is, upper variance control chart with lower reflecting barrier cl.
cu     for two-sided (sided="two") and fixed upper control limit (mode="fixed") a value larger than sigma0 has to been given, for all other cases cu is ignored.
hs     so-called headstart (enables fast initial response).
sided  distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at cl – the actual value of cl is not used), "Rupper" (upper chart with reflection at cl), "Rlower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.
mode    only deployed for sided="two" – with "fixed" an upper control limit (see cu) is set and only the lower is calculated to obtain the in-control ARL $L_0$, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
r       dimension of the resulting linear equation system (highest order of the collocation polynomials).
qm      number of quadrature nodes for calculating the collocation definite integrals.
qm.sigma number of quadrature nodes for convoluting the standard deviation uncertainty.
truncate size of truncated tail.
tail_approx controls whether the geometric tail approximation is used (is faster) or not.
c.error error bound for two succeeding values of the critical value during applying the secant rule.
a.error error bound for the quantile level alpha during applying the secant rule.

Details

sewma.crit.prerun determines the critical values (similar to alarm limits) for given in-control ARL $L_0$ by applying secant rule and using sewma.arl.prerun(). In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at sigma0. See Knoth (2010) for some details of the algorithm involved.
Value

Returns the lower and upper control limit cl and cu.

Author(s)

Sven Knoth

References


See Also

sewma.arl.prerun for calculation of ARL of variance charts under pre-run uncertainty and sewma.crit for the algorithm w/o pre-run uncertainty.

Examples

## will follow

---

**sewma.q**

_Calculate RL quantiles of EWMA (variance charts) control charts_

**Description**

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal variance.

**Usage**

```
sewma.q(l, cl, cu, sigma, df, alpha, hs=1, sided="upper", r=40, qm=30)
```

```
sewma.q.crit(l,L0,alpha,df,sigma0=1,cl=NULL,cu=NULL,hs=1,sided="upper",mode="fixed",ur=4,r=40,qm=30,c.error=1e-12,a.error=1e-9)
```

**Arguments**

- **l**
  - Smoothing parameter lambda of the EWMA control chart.

- **cl**
  - Deployed for sided="Rupper", that is, upper variance control chart with lower reflecting barrier cl.

- **cu**
  - For two-sided (sided="two") and fixed upper control limit (mode="fixed") a value larger than sigma0 has to be given, for all other cases cu is ignored.
Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann’s survival function iteration procedure. Thereby the ideas presented in Knoth (2007) are used. sewma.q.crit determines the critical values (similar to alarm limits) for given in-control RL quantile L0 at level alpha by applying secant rule and using sewma.sf(). In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the minimum of the cdf for given standard deviation is attained at sigma0.

Value

Returns a single value which resembles the RL quantile of order alpha and the lower and upper control limit cl and cu, respectively.

Author(s)

Sven Knoth
References


S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, Sequential Analysis 26, 251-264.


See Also

sewma.arl for calculation of ARL of variance charts and sewma.sf for the RL survival function.

Examples

## will follow

| Description | sewma.q.prerun Compute RL quantiles of EWMA (variance charts) control charts under pre-run uncertainty |

Usage

sewma.q.prerun(l, cl, cu, sigma, df1, df2, alpha, hs=1, sided="upper", r=40, qm=30, qm.sigma=30, truncate=1e-10)

sewma.q.crit.prerun(l, L0, alpha, df1, df2, sigma0=1, cl=NULL, cu=NULL, hs=1, sided="upper", mode="fixed", r=40, qm=30, qm.sigma=30, truncate=1e-10, tail_approx=TRUE, c.error=1e-10, a.error=1e-9)
Arguments

- **l**: smoothing parameter lambda of the EWMA control chart.
- **cl**: deployed for `sided="Rupper"`, that is, upper variance control chart with lower reflecting barrier `cl`.
- **cu**: for two-sided (`sided="two"`) and fixed upper control limit (`mode="fixed"`) a value larger than `sigma0` has to be given, for all other cases `cu` is ignored.
- **sigma, sigma0**: true and in-control standard deviation, respectively.
- **L0**: in-control quantile value.
- **alpha**: quantile level.
- **df1**: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one).
- **df2**: degrees of freedom of the pre-run variance estimator.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at `cl` – the actual value of `cl` is not used), "Rupper" (upper chart with reflection at `cl`), "Rlower" (lower chart with reflection at `cu`), and "two" (two-sided chart), respectively.
- **mode**: only deployed for `sided="two"` – with "fixed" an upper control limit (see `cu`) is set and only the lower is calculated to obtain the in-control ARL `L0`, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
- **r**: dimension of the resulting linear equation system (highest order of the collocation polynomials).
- **qm**: number of quadrature nodes for calculating the collocation definite integrals.
- **qm.sigma**: number of quadrature nodes for convoluting the standard deviation uncertainty.
- **truncate**: size of truncated tail.
- **tail_approx**: controls whether the geometric tail approximation is used (is faster) or not.
- **c.error**: error bound for two succeeding values of the critical value during applying the secant rule.
- **a.error**: error bound for the quantile level `alpha` during applying the secant rule.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann’s survival function iteration procedure. Thereby the ideas presented in Knoth (2007) are used. `sewma.q.crit.prerun` determines the critical values (similar to alarm limits) for given in-control RL quantile `L0` at level `alpha` by applying secant rule and using `sewma.sf()`. In case of `sided="two"` and `mode="unbiased"` a two-dimensional secant rule is applied that also ensures that the minimum of the cdf for given standard deviation is attained at `sigma0`. 
Value

Returns a single value which resembles the RL quantile of order \( \alpha \) and the lower and upper control limit \( cl \) and \( cu \), respectively.

Author(s)

Sven Knoth

References


See Also

sewma.q and sewma.q.crit for the version w/o pre-run uncertainty.

Examples

```r
## will follow
```

---

**sewma.sf**

*Compute the survival function of EWMA run length*

### Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal variance.

### Usage

```r
sewma.sf(n, l, cl, cu, sigma, df, hs=1, sided="upper", r=40, qm=30)
```

### Arguments

- `n` calculate sf up to value \( n \).
- `l` smoothing parameter lambda of the EWMA control chart.
- `cl` lower control limit of the EWMA control chart.
- `cu` upper control limit of the EWMA control chart.
- `sigma` true standard deviation.
- `df` actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.)
hs

so-called headstart (enables fast initial response).

sided
distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at $c_1$ – the actual value of $c_1$ is not used), "Rupper" (upper chart with reflection at $c_1$), "Rlower" (lower chart with reflection at $c_u$), and "two" (two-sided chart), respectively.

r
dimension of the resulting linear equation system (highest order of the collocation polynomials).

qm
number of quadrature nodes for calculating the collocation definite integrals.

Details

The survival function $P(L>n)$ and derived from it also the cdf $P(L<=n)$ and the pmf $P(L=n)$ illustrate the distribution of the EWMA run length. For large $n$ the geometric tail could be exploited. That is, with reasonable large $n$ the complete distribution is characterized. The algorithm is based on Waldmann’s survival function iteration procedure and on results in Knoth (2007).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References

S. Knoth (2007), Accurate ARL calculation for EWMA control charts monitoring simultaneously normal mean and variance, Sequential Analysis 26, 251-264.


See Also

sewma.arl for zero-state ARL computation of variance EWMA control charts.

Examples

## will follow
Compute the survival function of EWMA run length

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal variance.

Usage

sewma.sf.prerun(n, l, cl, cu, sigma, df1, df2, hs=1, sided="upper", qm=30, qm.sigma=30, truncate=1e-10, tail_approx=TRUE)

Arguments

- **n**: calculate sf up to value n.
- **l**: smoothing parameter lambda of the EWMA control chart.
- **cl**: lower control limit of the EWMA control chart.
- **cu**: upper control limit of the EWMA control chart.
- **sigma**: true standard deviation.
- **df1**: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- **df2**: degrees of freedom of the pre-run variance estimator.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided two-sided EWMA- $S^2$ control charts by choosing 'upper' (upper chart without reflection at cl – the actual value of cl is not used), 'Rupper' (upper chart with reflection at cl), 'Rlower' (lower chart with reflection at cu), and 'two' (two-sided chart), respectively.
- **qm**: number of quadrature nodes for calculating the collocation definite integrals.
- **qm.sigma**: number of quadrature nodes for convoluting the standard deviation uncertainty.
- **truncate**: size of truncated tail.
- **tail_approx**: Controls whether the geometric tail approximation is used (is faster) or not.

Details

The survival function $P(L>n)$ and derived from it also the cdf $P(L<=n)$ and the pmf $P(L=n)$ illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann’s survival function iteration procedure and on results in Knoth (2007)...

Value

Returns a vector which resembles the survival function up to a certain point.
Author(s)
Sven Knoth

References

See Also
sewma.sf for the RL survival function of EWMA control charts w/o pre-run uncertainty.

Examples
## will follow

---

tewma.arl  Compute ARLs of Poisson TEWMA control charts

Description
Computation of the (zero-state) Average Run Length (ARL) at given Poisson mean mu.

Usage
tewma.arl(lambda, k, lk, uk, mu, z0, rando=FALSE, gl=0, gu=0)

Arguments

- lambda: smoothing parameter of the EWMA p control chart.
- k: resolution of grid (natural number).
- lk: lower control limit of the TEWMA control chart, integer.
- uk: upper control limit of the TEWMA control chart, integer.
- mu: mean value of Poisson distribution.
- z0: so-called headstart (give fast initial response) – it is proposed to use the in-control mean.
- rando: Distinguish between control chart design without or with randomisation. In the latter case some meaningful values for gl and gu should be provided.
- gl: randomisation probability at the lower limit.
- gu: randomisation probability at the upper limit.
Details

A new idea of applying EWMA smoothing to count data. Here, the thinning operation is applied to independent Poisson variates is performed. Moreover, the original thinning principle is expanded to multiples of one over k to allow finer grids and finally better detection performance. It is highly recommended to read the corresponding paper (see below).

Value

Return single value which resemble the ARL.

Author(s)

Sven Knoth

References


See Also

later.

Examples

```r
# MWK (2018)
lambda <- 0.1 # (T)EWMA smoothing constant
mu0 <- 5 # in-control mean
k <- 10 # resolution
z0 <- round(k*mu0) # starting value of (T)EWMA sequence
# (i) without randomisation
lk <- 28
uk <- 75
L0 <- tewma.arl(lambda, k, lk, uk, mu0, z0)
# should be 501.9703
# (ii) with randomisation
uk <- 76 # lk is not changed
gl <- 0.5446310
gu <- 0.1375617
L0 <- tewma.arl(lambda, k, lk, uk, mu0, z0, rando=TRUE, gl=gl, gu=gu)
# should be 500
```

Description

For constructing tolerance intervals, which cover a given proportion \( p \) of a normal distribution with unknown mean and variance with confidence \( 1 - \alpha \), one needs to calculate the so-called tolerance limit factors \( k \). These values are computed for a given sample size \( n \).
tol.lim.fac

Usage

tol.lim.fac(n,p,a,mode="WW",m=30)

Arguments

n  sample size.
p  coverage.
a  error probability $\alpha$, resulting interval covers at least proportion $p$ with confidence of at least $1 - \alpha$.
mode  distinguish between Wald/Wolfowitz’ approximation method ("WW") and the more accurate approach ("exact") based on Gauss-Legendre quadrature.
m  number of abscissas for the quadrature (needed only for method="exact"), of course, the larger the more accurate.

Details

tol.lim.fac determines tolerance limits factors $k$ by means of the fast and simple approximation due to Wald/Wolfowitz (1946) and of Gauss-Legendre quadrature like Odeh/Owen (1980), respectively, who used in fact the Simpson Rule. Then, by $\bar{x} \pm k \cdot s$ one can build the tolerance intervals which cover at least proportion $p$ of a normal distribution for given confidence level of $1 - \alpha$. $\bar{x}$ and $s$ stand for the sample mean and the sample standard deviation, respectively.

Value

Returns a single value which resembles the tolerance limit factor.

Author(s)

Sven Knoth

References


See Also

*qnorm* for the "asymptotic" case – cf. second example.

Examples

```r
n <- 2:10
p <- .95
a <- .05
kWW <- sapply(n,p,a=tol.lim.fac)
kEX <- sapply(n,p,a=a,mode="exact",tol.lim.fac)
```
print(cbind(n,kWW,kEX),digits=4)
## Odeh/Owen (1980), page 98, in Table 3.4.1
## n factor k
## 2 36.519
## 3 9.789
## 4 6.341
## 5 5.077
## 6 4.422
## 7 4.020
## 8 3.746
## 9 3.546
## 10 3.393

## n -> infty
n <- 10^(1:7)
p <- .95
a <- .05
kEX <- round(sapply(n,p=p,a=a,mode="exact",tol.lim.fac),digits=4)
kEXinf <- round(qnorm(1-a/2),digits=4)
print(rbind(cbind(n,kEX),c("infinity",kEXinf)),quote=FALSE)

x.res.ewma.arl

---

**x.res.ewma.arl**  
*Compute ARLs of EWMA residual control charts*

**Description**

Computation of the (zero-state) Average Run Length (ARL) for EWMA residual control charts monitoring normal mean, variance, or mean and variance simultaneously. Additionally, the probability of misleading signals (PMS) is calculated.

**Usage**

```r
x.res.ewma.arl(l, c, mu, alpha=0, n=5, hs=0, r=40)
s.res.ewma.arl(l, cu, sigma, mu=0, alpha=0, n=5, hs=1, r=40, qm=30)
x.s.res.ewma.arl(lx, cx, ls, csu, mu, sigma, alpha=0, n=5, hsx=0, rx=40, hss=1, rs=40, qm=30)
x.s.res.ewma.pms(lx, cx, ls, csu, mu, sigma, type="3", alpha=0, n=5, hsx=0, rx=40, hss=1, rs=40, qm=30)
```

**Arguments**

- `l, lx, ls`  
  smoothing parameter(s) lambda of the EWMA control chart.
- `c, cu, cx, csu`  
  critical value (similar to alarm limit) of the EWMA control charts.
- `mu`  
  true mean.
- `sigma`  
  true standard deviation.
alpha the AR(1) coefficient – first order autocorrelation of the original data.

n batch size.

hs, hsx, hss so-called headstart (enables fast initial response).

r, rx, rs number of quadrature nodes or size of collocation base, dimension of the resulting linear equation system is equal to r (two-sided).

qm number of nodes for collocation quadratures.

type PMS type, for PMS="3" (the default) the probability of getting a mean signal despite the variance changed, and for PMS="4" the opposite case is dealt with.

Details

The above list of functions provides the application of algorithms developed for iid data to the residual case. To be more precise, the underlying model is a sequence of normally distributed batches with size n with autocorrelation within the batch and independence between the batches (see also the references below). It is restricted to the classical EWMA chart types, that is two-sided for the mean, upper charts for the variance, and all equipped with fixed limits. The autocorrelation is modeled by an AR(1) process with parameter alpha. Additionally, with xs.res.ewma.pms the probability of misleading signals (PMS) of type is calculated. This is offered exclusively in this small collection so that for iid data this function has to be used too (with alpha=0).

Value

Return single values which resemble the ARL and the PMS, respectively.

Author(s)

Sven Knoth

References


See Also

xewma.arl, sewma.arl, and xsewma.arl as more elaborated functions in the iid case.
Examples

```r
## Not run:

cat("\nFragments of Table 2 (n=5, lambda.1=lambda.2)\n")

lambdas <- c(.5, .25, .1, .05)
L0 <- 500
n <- 5

crit <- NULL
for ( lambda in lambdas ) {
  cs <- xsewma.crit(lambda, lambda, L0, n-1)
  x.e <- round(cs[1], digits=4)
  names(x.e) <- NULL
  s.e <- round((cs[3]-1) * sqrt((2-lambda)/lambda)*sqrt((n-1)/2), digits=4)
  names(s.e) <- NULL
  crit <- rbind(crit, data.frame(lambda, x.e, s.e))
}

## original values are (Markov chain approximation with 50 states)
# lambda x.e s.e
# 0.50 3.2765 4.6439
# 0.25 3.2168 4.0149
# 0.10 3.0578 3.3376
# 0.05 2.8817 2.9103

print(crit)

cat("\nFragments of Table 4 (n=5, lambda.1=lambda.2=0.1)\n")

lambda <- .1
# the algorithm used in Knoth/Schmid is less accurate -- proceed with their values
cx <- x.e <- 3.0578
s.e <- 3.3376
csu <- 1 + s.e * sqrt(lambda/(2-lambda))*sqrt(2/(n-1))
alpha <- .3
a.values <- c((0:6)/4, 2)
d.values <- c(1 + (0:5)/10, 1.75 , 2)

arls <- NULL
for ( delta in d.values ) {
  row <- NULL
  for ( mu in a.values ) {
    arl <- round(xs.res.ewma.arl(lambda, cx, lambda, csu, mu*sqrt(n), delta, alpha=alpha, n=n),
                    digits=2)
    names(arl) <- NULL
    row <- c(row, arl)
  }
  row <- c(row, arl)
}
```
x.res.ewma.arl

```r
  arls <- rbind(arls, data.frame(t(row)))
  names(arls) <- a.values
  rownames(arls) <- d.values

  # Original values are (now Monte-Carlo with 10^6 replicates)
  #
  # 0 0.25 0.5 0.75 1 1.25 1.5 2
  # 1 502.44 49.50 14.21 7.93 5.53 4.28 3.53 2.65
  # 1.1 73.19 32.91 13.33 7.82 5.52 4.29 3.54 2.66
  # 1.2 24.42 18.88 11.37 7.44 5.42 4.27 3.54 2.67
  # 1.3 13.11 11.83 9.09 6.74 5.18 4.17 3.50 2.66
  # 1.4 8.74 8.31 7.19 5.89 4.81 4.00 3.41 2.64
  # 1.5 6.50 6.31 5.80 5.08 4.37 3.76 3.28 2.59
  # 1.75 3.94 3.90 3.78 3.59 3.35 3.09 2.83 2.40
  # 2 2.85 2.84 2.80 2.73 2.63 2.51 2.39 2.14

  print(arls)


  cat("\nFragments of Table 5 (n=5, lambda=0.1)\n")

  d.values <- c(1.02, 1 + (1:5)/10, 1.75 , 2)

  arl.x <- arl.s <- arl.xs <- PMS.3 <- NULL
  for (delta in d.values) {
    arl.x <- c(arl.x, round(x.res.ewma.arl(lambda, cx/delta, 0, n=n),
                             digits=3))
    arl.s <- c(arl.s, round(s.res.ewma.arl(lambda, csu, delta, n=n),
                             digits=3))
    arl.xs <- c(arl.xs, round(xs.res.ewma.arl(lambda, cx, lambda, csu, 0, delta, n=n),
                                digits=3))
    PMS.3 <- c(PMS.3, round(xs.res.ewma.pms(lambda, cx, lambda, csu, 0, delta, n=n),
                                digits=6))
  }

  # Original values are (Markov chain approximation)
  #
  #   delta  arl.x  arl.s  arl.xs  PMS.3
  # 1 1.02 833.086 518.935 323.324 0.381118
  # 1.10 454.101 84.208 73.029 0.145005
  # 1.20 250.665 25.871 24.432 0.071024
  # 1.30 157.343 13.567 13.125 0.047193
  # 1.40 108.112 9.841 9.734 0.035945
  # 1.50 79.308 6.614 6.493 0.029499
  # 1.75 44.128 3.995 3.942 0.021579
  # 2.00 28.974 2.887 2.853 0.018220

  print(cbind(delta=d.values, arl.x, arl.s, arl.xs, PMS.3))

  cat("\nFragments of Table 6 (n=5, lambda=0.1)\n")
```
alphas <- c(-0.9, -0.5, -0.3, 0, 0.3, 0.5, 0.9)
deltas <- c(0.05, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 2)

PMS.4 <- NULL
for ( ir in 1:length(deltas) ) {
  mu <- deltas[ir]*sqrt(n)
  pms <- NULL
  for ( alpha in alphas ) {
    pms <- c(pms, round(xs.res.ewma.pms(lambda, cx, lambda, csu, mu, 1, type="4", alpha=alpha, n=n),
      digits=6))
  }
  PMS.4 <- rbind(PMS.4, data.frame(delta=deltas[ir], t(pms)))
}

names(PMS.4) <- c("delta", alphas)
rownames(PMS.4) <- NULL

## original values are (Markov chain approximation)
# delta  -0.9  -0.5  -0.3   0   0.3   0.5   0.9
#  0.05  0.055789  0.224521  0.279842  0.342805  0.41915  0.471386
#  0.25  0.003566  0.009522  0.014580  0.025786  0.044892  0.066584  0.192023
#  0.50  0.002994  0.001816  0.002596  0.004774  0.009259  0.015303  0.072945
#  0.75  0.006967  0.000703  0.000837  0.001529  0.003400  0.006424  0.046602
#  1.00  0.005098  0.000402  0.000370  0.000625  0.001589  0.003490  0.039978
#  1.25  0.000084  0.000266  0.000202  0.000300  0.000867  0.002220  0.039773
#  1.50  0.000000  0.000256  0.000120  0.000163  0.000531  0.001584  0.042734
#  2.00  0.000000  0.000311  0.000091  0.000056  0.000259  0.001029  0.054543

print(PMS.4)

## End(Not run)

xcusum.ad

Compute steady-state ARLs of CUSUM control charts

Description

Computation of the steady-state Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

Usage

xcusum.ad(k, h, mu1, mu0 = 0, sided = "one", r = 30)

Arguments

- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **mu1**: out-of-control mean.
xcusum.ad

mu0 in-control mean.
sided distinguish between one-, two-sided and Crosier's modified two-sided CUSUM scheme by choosing "one", "two", and "Crosier", respectively.
r number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-, two-sided) or 2r+1 (Crosier).

Details

xcusum.ad determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.

Value

Returns a single value which resembles the steady-state ARL.

Note

Be cautious in increasing the dimension parameter r for two-sided CUSUM schemes. The resulting matrix dimension is r^2 times r^2. Thus, go beyond 30 only on fast machines. This is the only case, were the package routines are based on the Markov chain approach. Moreover, the two-sided CUSUM scheme needs a two-dimensional Markov chain.

Author(s)

Sven Knoth

References


See Also

xcusum.arl for zero-state ARL computation and xewma.ad for the steady-state ARL of EWMA control charts.

Examples

```r
## comparison of zero-state (= worst case ) and steady-state performance
## for one-sided CUSUM control charts

k <- .5
h <- xcusum.crit(k,500)
mu <- c(0,.5,1,1.5,2)
arl <- sapply(mu,k=k,h=h,xcusum.arl)
ad <- sapply(mu,k=k,h=h,xcusum.ad)
round(cbind(mu,arl,ad),digits=2)
```
## Crosier (1986), Crosier's modified two-sided CUSUM

He introduced the modification and evaluated it by means of Markov chain approximation

```r
k <- .5
h2 <- 4
hC <- 3.73
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
ad2 <- sapply(mu,k=k,h=h2,sided="two",r=20,xcusum.ad)
adC <- sapply(mu,k=k,h=hC,sided="Crosier",xcusum.ad)
round(cbind(mu,ad2,adC),digits=2)
```

## results in the original paper are (in Table 5)

<table>
<thead>
<tr>
<th>mu</th>
<th>ad2</th>
<th>adC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>163.</td>
<td>164.</td>
</tr>
<tr>
<td>0.25</td>
<td>71.6</td>
<td>69.0</td>
</tr>
<tr>
<td>0.50</td>
<td>25.2</td>
<td>24.3</td>
</tr>
<tr>
<td>0.75</td>
<td>12.3</td>
<td>12.1</td>
</tr>
<tr>
<td>1.00</td>
<td>7.68</td>
<td>7.69</td>
</tr>
<tr>
<td>1.50</td>
<td>4.31</td>
<td>4.39</td>
</tr>
<tr>
<td>2.00</td>
<td>3.03</td>
<td>3.12</td>
</tr>
<tr>
<td>2.50</td>
<td>2.38</td>
<td>2.46</td>
</tr>
<tr>
<td>3.00</td>
<td>2.00</td>
<td>2.07</td>
</tr>
<tr>
<td>4.00</td>
<td>1.55</td>
<td>1.60</td>
</tr>
<tr>
<td>5.00</td>
<td>1.22</td>
<td>1.29</td>
</tr>
</tbody>
</table>

### xcusum.arl

**Compute ARLs of CUSUM control charts**

#### Description

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

#### Usage

```r
xcusum.arl(k, h, mu, hs = 0, sided = "one", method = "igl", q = 1, r = 30)
```

#### Arguments

- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **mu**: true mean.
- **hs**: so-called headstart (give fast initial response).
- **sided**: distinguish between one-, two-sided and Crosier's modified two-sided CUSUM scheme by choosing "one", "two", and "Crosier", respectively.
- **method**: deploy the integral equation ("igl") or Markov chain approximation ("mc") method to calculate the ARL (currently only for two-sided CUSUM implemented).
change point position. For $q = 1$ and $\mu = \mu_0$ and $\mu = \mu_1$, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For $q > 1$ and $\mu = 0$ conditional delays, that is, $E_q(L - q + 1 | L \geq q)$, will be determined. Note that $\mu_0=0$ is implicitly fixed.

number of quadrature nodes, dimension of the resulting linear equation system is equal to $r+1$ (one-, two-sided) or $2r+1$ (Crosier).

Details

cusum.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a vector of length $q$ which resembles the ARL and the sequence of conditional expected delays for $q=1$ and $q>1$, respectively.

Author(s)

Sven Knoth

References


D. Brook, D. A. Evans (1972), An approach to the probability distribution of cusum run length, Biometrika 59, 539-548.


See Also

cusum.arl for zero-state ARL computation of EWMA control charts and cusum.ad for the steady-state ARL.

Examples

```r
## Brook/Evans (1972), one-sided CUSUM
## Their results are based on the less accurate Markov chain approach.

k <- .5
h <- 3
```
round(c( xcusum.arl(k,h,0), xcusum.arl(k,h,1.5) ),digits=2)

## results in the original paper are L0 = 117.59, L1 = 3.75 (in Subsection 4.3).

## Lucas, Crosier (1982)
## (one- & two-sided CUSUM with possible headstarts

k <- .5
h <- 4
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
arl1 <- sapply(mu,k=k,h=h,sided="two",xcusum.arl)
arl2 <- sapply(mu,k=k,h=h,hs=h/2,sided="two",xcusum.arl)
round(cbind(mu,arl1,arl2),digits=2)

## results in the original paper are (in Table 1)
## 0.00 168. 149.
## 0.25 74.2 62.7
## 0.50 26.6 20.1
## 0.75 13.3 8.97
## 1.00 8.38 5.29
## 1.50 4.75 2.86
## 2.00 3.34 2.01
## 2.50 2.62 1.59
## 3.00 2.19 1.32
## 4.00 1.71 1.07
## 5.00 1.31 1.01

## Vance (1986), one-sided CUSUM
## The first paper on using Nystroem method and Gauss-Legendre quadrature
## for solving the ARL integral equation (see as well Goel/Wu, 1971)

k <- 0
h <- 10
mu <- c(-.25,-.125,0,.125,.25,.5,.75,1)
round(cbind(mu,sapply(mu,k=k,h=h,xcusum.arl)),digits=2)

## results in the original paper are (in Table 1 incl. Goel/Wu (1971) results)
## -0.25 2071.51
## -0.125 400.28
## 0.0 124.66
## 0.125 59.30
## 0.25 36.71
## 0.5 20.37
## 0.75 14.06
## 1.0 10.75

## Waldmann (1986),
## one- & two-sided CUSUM

## one-sided case

k <- .5
h <- 3
\texttt{xcusum.arl}

\begin{verbatim}
mu <- c(-.5,0,.5)
round(sapply(mu,k=k,h=h,xcusum.arl),digits=2)

## results in the original paper are 1963, 117.4, and 17.35, resp.
## (in Tables 3, 1, and 5, resp.).

## two-sided case

k <- .6
h <- 3
round(xcusum.arl(k,h,-.2,sided="two"),digits=1) # fits to Waldmann's setup

## result in the original paper is 65.4 (in Table 6).

## Crosier (1986), Crosier's modified two-sided CUSUM
## He introduced the modification and evaluated it by means of
## Markov chain approximation

k <- .5
h <- 3.73
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,4,5)
round(cbind(mu,sapply(mu,k=k,h=h,sided="Crosier",xcusum.arl)),digits=2)

## results in the original paper are (in Table 3)
## 0.00 168.
## 0.25 70.7
## 0.50 25.1
## 0.75 12.5
## 1.00 7.92
## 1.50 4.49
## 2.00 3.17
## 2.50 2.49
## 3.00 2.09
## 4.00 1.60
## 5.00 1.22

## SAS/QC manual 1999
## one- and two-sided CUSUM schemes

## one-sided

k <- .25
h <- 8
mu <- 2.5
print(xcusum.arl(k,h,mu),digits=12)
print(xcusum.arl(k,h,mu,hs=.1),digits=12)

## original results are 4.1500836225 and 4.1061588131.

## two-sided

print(xcusum.arl(k,h,mu,sided="two"),digits=12)
\end{verbatim}
## original result is 4.1500826715.

### xcusum.crit

**Compute decision intervals of CUSUM control charts**

### Description

Computation of the decision intervals (alarm limits) for different types of CUSUM control charts monitoring normal mean.

### Usage

```r
cusum.crit(k, L0, mu0 = 0, hs = 0, sided = "one", r = 30)
```

### Arguments

- **k**: reference value of the CUSUM control chart.
- **L0**: in-control ARL.
- **mu0**: in-control mean.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one-, two-sided and Crosier’s modified two-sided CUSUM scheme by choosing "one", "two", and "Crosier", respectively.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-, two-sided) or \( 2r+1 \) (Crosier).

### Details

`xcusum.crit` determines the decision interval (alarm limit) for given in-control ARL \( L_0 \) by applying secant rule and using `xcusum.arl()`.

### Value

Returns a single value which resembles the decision interval \( h \).

### Author(s)

Sven Knoth

### See Also

`xcusum.arl` for zero-state ARL computation.

### Examples

```r
k <- .5
incontrolARL <- c(500, 5000, 50000)
sapply(incontrolARL, k, xcusum.crit, r=10) # accuracy with 10 nodes
sapply(incontrolARL, k, xcusum.crit, r=20) # accuracy with 20 nodes
sapply(incontrolARL, k, xcusum.crit) # accuracy with 30 nodes
```
Compute the CUSUM reference value \( k \) for given in-control ARL and threshold \( h \)

Description

Computation of the reference value \( k \) for one-sided CUSUM control charts monitoring normal mean, if the in-control ARL \( L_0 \) and the alarm threshold \( h \) are given.

Usage

\[	exttt{xcusum.crit.L0h}(L_0, h, hs=0, sided="one", r=30, L0.eps=1e-6, k.eps=1e-8)
\]

Arguments

- **L0**: in-control ARL.
- **h**: alarm level of the CUSUM control chart.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one-, two-sided and Crosier’s modified two-sided CUSUM scheme choosing "one", "two", and "Crosier", respectively.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-, two-sided) or \( 2r+1 \) (Crosier).
- **L0.eps**: error bound for the \( L_0 \) error.
- **k.eps**: bound for the difference of two successive values of \( k \).

Details

\texttt{xcusum.crit.L0h} determines the reference value \( k \) for given in-control ARL \( L_0 \) and alarm level \( h \) by applying secant rule and using \texttt{xcusum.arl}(). Note that not for any combination of \( L_0 \) and \( h \) a solution exists – for given \( L_0 \) there is a maximal value for \( h \) to get a valid result \( k \).

Value

Returns a single value which resembles the reference value \( k \).

Author(s)

Sven Knoth

See Also

\texttt{xcusum.arl} for zero-state ARL computation.
Examples

```r
L0 <- 100
h.max <- xcusum.crit(0, L0, 0)
hs <- (300:1)/100
hs <- hs[hs < h.max]
ks <- NULL
for ( h in hs ) ks <- c(ks, xcusum.crit.L0h(L0, h))
k.max <- qnorm( 1 - 1/L0 )
plot(hs, ks, type="l", ylim=c(0, max(k.max, ks)), xlab="h", ylab="k")
abline(h=c(0, k.max), col="red")
```

xcusum.crit.L0L1  
Compute the CUSUM k and h for given in-control ARL L0 and out-of-control L1

Description

Computation of the reference value k and the alarm threshold h for one-sided CUSUM control charts monitoring normal mean, if the in-control ARL L0 and the out-of-control L1 are given.

Usage

```r
xcusum.crit.L0L1(L0, L1, hs=0, sided="one", r=30, L1.eps=1e-6, k.eps=1e-8)
```

Arguments

- `L0` in-control ARL.
- `L1` out-of-control ARL.
- `hs` so-called headstart (enables fast initial response).
- `sided` distinguishes between one-, two-sided and Crosier’s modified two-sided CUSUM schemoosing "one","two", and "Crosier", respectively.
- `r` number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-, two-sided) or 2r+1 (Crosier).
- `L1.eps` error bound for the L1 error.
- `k.eps` bound for the difference of two successive values of k.

Details

`xcusum.crit.L0L1` determines the reference value k and the alarm threshold h for given in-control ARL L0 and out-of-control ARL L1 by applying secant rule and using `xcusum.arl()` and `xcusum.crit()`. These CUSUM design rules were firstly (and quite rarely afterwards) used by Ewan and Kemp.

Value

Returns two values which resemble the reference value k and the threshold h.
Author(s)
Sven Knoth

References
W. D. Ewan and K. W. Kemp (1960), Sampling inspection of continuous processes with no auto-
correlation between successive results, *Biometrika* 47, 363-380.

K. W. Kemp (1962), The Use of Cumulative Sums for Sampling Inspection Schemes, *Journal of

See Also

xcusum.arl for zero-state ARL and xcusum.crit for threshold h computation.

Examples

```r
## Table 2 from Ewan/Kemp (1960) -- one-sided CUSUM
#
# A.R.L. at A.Q.L. A.R.L. at A.Q.L. k h
# 1000 3 1.12 2.40
# 1000 7 0.65 4.06
# 500 3 1.04 2.26
# 500 7 0.60 3.80
# 250 3 0.94 2.11
# 250 7 0.54 3.51
#
L0.set <- c(1000, 500, 250)
L1.set <- c(3, 7)
cat("\nL0	L1	k	h
")
for ( L0 in L0.set ) {
  for ( L1 in L1.set ) {
    result <- round(xcusum.crit.L0L1(L0, L1), digits=2)
    cat(paste(L0, L1, result[1], result[2], sep="\t"), "\n")
  }
}
#
# two confirmation runs
xcusum.arl(0.54, 3.51, 0) # Ewan/Kemp
xcusum.arl(result[1], result[2], 0) # here
xcusum.arl(0.54, 3.51, 2*0.54) # Ewan/Kemp
xcusum.arl(result[1], result[2], 2*result[1]) # here
#
## Table II from Kemp (1962) -- two-sided CUSUM
#
# Lr k
# La=250 La=500 La=1000
# 2.5 1.05 1.17 1.27
# 3.0 0.94 1.035 1.13
# 4.0 0.78 0.85 0.92
# 5.0 0.68 0.74 0.80
# 6.0 0.60 0.655 0.71
```
xcusum.q

Compute RL quantiles of CUSUM control charts

Description

Computation of quantiles of the Run Length (RL) for CUSUM control charts monitoring normal mean.

Usage

xcusum.q(k, h, mu, alpha, hs=0, sided="one", r=40)

Arguments

k reference value of the CUSUM control chart.

h decision interval (alarm limit, threshold) of the CUSUM control chart.

mu true mean.

alpha quantile level.

hs so-called headstart (enables fast initial response).

sided distinguishes between one- and two-sided CUSUM control chart by choosing "one" and "two", respectively.

r number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1.

Details

Instead of the popular ARL (Average Run Length) quantiles of the CUSUM stopping time (Run Length) are determined. The algorithm is based on Waldmann’s survival function iteration procedure.

Value

Returns a single value which resembles the RL quantile of order q.
Author(s)
Sven Knoth

References

See Also
xcusum.arl for zero-state ARL computation of CUSUM control charts.

Examples
```r
## Waldmann (1986), one-sided CUSUM, Table 2
## original values are 345, 82, 9
XCUSUM.Q <- Vectorize("xcusum.q", "alpha")
k <- .5
h <- 3
mu <- 0 # corresponds to Waldmann's -0.5
a.list <- c(.95, .5, .05)
rl.quantiles <- ceiling(XCUSUM.Q(k, h, mu, a.list))
cbind(a.list, rl.quantiles)
```

**xCusum.sf**

*Compute the survival function of CUSUM run length*

Description
Computation of the survival function of the Run Length (RL) for CUSUM control charts monitoring normal mean.

Usage
```r
xcusum.sf(k, h, mu, n, hs=0, sided="one", r=40)
```

Arguments
- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **mu**: true mean.
- **n**: calculate sf up to value n.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided CUSUM control chart by choosing "one" and "two", respectively.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1.
Details

The survival function \( P(L>n) \) and derived from it also the cdf \( P(L<=n) \) and the pmf \( P(L=n) \) illustrate the distribution of the CUSUM run length. For large \( n \) the geometric tail could be exploited. That is, with reasonable large \( n \) the complete distribution is characterized. The algorithm is based on Waldmann’s survival function iteration procedure.

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References


See Also

`xcusum.q` for computation of CUSUM run length quantiles.

Examples

```r
## Waldmann (1986), one-sided CUSUM, Table 2

k <- .5
h <- 3
mu <- 0 # corresponds to Waldmann's -0.5
SF <- xcusum.sf(k, h, 0, 1000)
plot(1:length(SF), SF, type="l", xlab="n", ylab="P(L>n)", ylim=c(0,1))
```

---

**xCusum.arl**

*Compute ARLs of CUSUM control charts under drift*

Description

Computation of the (zero-state and other) Average Run Length (ARL) under drift for one-sided CUSUM control charts monitoring normal mean.

Usage

```r
xCusum.arl(k, h, delta, hs = 0, sided = "one", mode = "Gan", m = NULL, q = 1, r = 30, with0 = FALSE)
```
Arguments

- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **delta**: true drift parameter.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided CUSUM control chart by choosing "one" and "two", respectively. Currently, the two-sided scheme is not implemented.
- **mode**: decide whether Gan's or Knoth's approach is used. Use "Gan" and "Knoth", respectively.
- **m**: parameter used if mode="Gan". \( m \) is design parameter of Gan’s approach. If \( m=\text{NULL} \), then \( m \) will increased until the resulting ARL does not change anymore.
- **q**: change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu_1 = 0 \) conditional delays, that is, \( E_q(L - q + 1|L \geq) \), will be determined. Note that \( \mu_0=0 \) is implicitly fixed. Deploy large \( q \) to mimic steady-state. It works only for mode="knoth".
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-sided) or \( r \) (two-sided).
- **with0**: defines whether the first observation used for the RL calculation follows already \( 1*\text{delta} \) or still \( 0*\text{delta} \). With \( q \) additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for one-sided CUSUM control charts under drift. In case of Gan’s framework, the usual ARL function with \( \mu=m*\text{delta} \) is determined and recursively via \( m-1, m-2, \ldots, 1 \) (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers. Note that two-sided CUSUM charts under drift are difficult to treat.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

F. F. Gan (1992), CUSUM control charts under linear drift, Statistician 41, 71-84.
S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.


See Also

`x dusum.arl` and `x dusum.ad` for zero-state and steady-state ARL computation of CUSUM control charts for the classical step change model.

Examples

```r
# Gan (1992)
# Table 1
# original values are
# delta arl
# 0.0001 475
# 0.0005 261
# 0.0010 187
# 0.0020 129
# 0.0050 76.3
# 0.0100 52.0
# 0.0200 35.2
# 0.0500 21.4
# 0.1000 15.0
# 0.5000 6.95
# 1.0000 5.16
# 3.0000 3.30
# Not run: k <- .25
# h <- 8
# r <- 50
DxDcusum.arl <- Vectorize(xDcusum.arl, "delta")
deltas <- c(0.0001, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.5, 1, 3)
arl.like.Gan <-
  round(DxDcusum.arl(k, h, deltas, r=r, with0=TRUE), digits=2)
arl.like.Knoth <-
  round(DxDcusum.arl(k, h, deltas, r=r, mode="Knoth", with0=TRUE), digits=2)
data.frame(deltas, arl.like.Gan, arl.like.Knoth)
# End(Not run)

# Zou et al. (2009)
# Table 1
# original values are
# delta arl1 arl2 arl3
# 0 1730
# 0.0005 345 412 470
# 0.001 231 275 317
# 0.005 86.6 98.6 112
# 0.01 56.9 61.8 69.3
# 0.05 22.6 21.6 22.7
# 0.1 15.4 14.7 14.2
# 0.5 6.60 5.54 5.17
```

```
# 1.0  4.63  3.80  3.45
# 2.0  3.17  2.67  2.32
# 3.0  2.79  2.04  1.96
# 4.0  2.10  1.98  1.74
## Not run:
k1 <- 0.25
k2 <- 0.5
k3 <- 0.75
h1 <- 9.660
h2 <- 5.620
h3 <- 3.904
deltas <- c(0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1:4)
arl1 <- c(round(xcusum.arl(k1, h1, 0, r=r), digits=2),
          round(DxDcusum.arl(k1, h1, deltas, r=r), digits=2))
arl2 <- c(round(xcusum.arl(k2, h2, 0), digits=2),
          round(DxDcusum.arl(k2, h2, deltas, r=r), digits=2))
arl3 <- c(round(xcusum.arl(k3, h3, 0, r=r), digits=2),
          round(DxDcusum.arl(k3, h3, deltas, r=r), digits=2))
data.frame(delta=c(0, deltas), arl1, arl2, arl3)
## End(Not run)

---

**xDewma.arl**

*Compute ARLs of EWMA control charts under drift*

**Description**

Computation of the (zero-state and other) Average Run Length (ARL) under drift for different types of EWMA control charts monitoring normal mean.

**Usage**

```r
xDewma.arl(l, c, delta, zr = 0, hs = 0, sided = "one", limits = "fix",
           mode = "Gan", m = NULL, q = 1, r = 40, with0 = FALSE)
```

**Arguments**

- `l` smoothing parameter lambda of the EWMA control chart.
- `c` critical value (similar to alarm limit) of the EWMA control chart.
- `delta` true drift parameter.
- `zr` reflection border for the one-sided chart.
- `hs` so-called headstart (enables fast initial response).
- `sided` distinguish between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
- `limits` distinguishes between different control limits behavior.
- `mode` decide whether Gan’s or Knoth’s approach is used. Use "Gan" and "Knoth", respectively.
m  parameter used if mode="Gan". m is design parameter of Gan’s approach. If m=NULL, then m will increased until the resulting ARL does not change anymore.

q  change point position. For q = 1 and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1|L \geq) \), will be determined. Note that mu0=0 is implicitly fixed. Deploy large q to mimic steady-state. It works only for mode="Knoth".

r  number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).

with\( \theta \)  defines whether the first observation used for the RL calculation follows already 1*delta or still 0*delta. With q additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for EWMA control charts under drift. In case of Gan’s framework, the usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

xewma.arl and xewma.ad for zero-state and steady-state ARL computation of EWMA control charts for the classical step change model.
Examples

```r
# Not run:
DxDewma.arl <- Vectorize(xDewma.arl, "delta")
# Gan (1991)
# Table 1
# original values are
# delta arlE1 arlE2 arlE3
# 0 500 500 500
# 0.0001 482 460 424
# 0.0010 289 231 185
# 0.0020 210 162 129
# 0.0050 126 94.6 77.9
# 0.0100 81.7 61.3 52.7
# 0.0500 27.5 21.8 21.9
# 0.1000 17.0 14.2 15.3
# 1.0000 4.09 4.28 5.25
# 3.0000 2.60 2.90 3.43
#
# lambda1 <- 0.676
# lambda2 <- 0.242
# lambda3 <- 0.047
# h1 <- 2.204/sqrt(lambda1/(2-lambda1))
# h2 <- 1.111/sqrt(lambda2/(2-lambda2))
# h3 <- 0.403/sqrt(lambda3/(2-lambda3))
# deltas <- c(.0001, .001, .002, .005, .01, .05, .1, 1, 3)
# arlE10 <- round(xewma.arl(lambda1, h1, 0, sided="two"), digits=2)
# arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, h1, deltas, sided="two", with0=TRUE),
# digits=2))
# arlE20 <- round(xewma.arl(lambda2, h2, 0, sided="two"), digits=2)
# arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, h2, deltas, sided="two", with0=TRUE),
# digits=2))
# arlE30 <- round(xewma.arl(lambda3, h3, 0, sided="two"), digits=2)
# arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, h3, deltas, sided="two", with0=TRUE),
# digits=2))
# data.frame(delta=c(0, deltas), arlE1, arlE2, arlE3)
#
## do the same with more digits for the alarm threshold
L0 <- 500
h1 <- xewma.crit(lambda1, L0, sided="two")
# lambda2 <- xewma.crit(lambda2, L0, sided="two")
# lambda3 <- xewma.crit(lambda3, L0, sided="two")
# hs <- c(h1, h2, h3) * sqrt(lambdas/(2-lambdas))
# compare with Gan's values 2.204, 1.111, 0.403
round(hs, digits=3)
#
arlE10 <- round(xewma.arl(lambda1, h1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, h1, deltas, sided="two", with0=TRUE),
# digits=2))
arlE20 <- round(xewma.arl(lambda2, h2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, h2, deltas, sided="two", with0=TRUE),
# digits=2))
```

```r
arlE30 <- round(xewma.arl(lambda3, h3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, h3, deltas, sided="two", with0=TRUE), digits=2))
data.frame(delta=c(0, deltas), arlE1, arlE2, arlE3)

## Aerne et al. (1991) -- two-sided EWMA
## Table I (continued)
## original numbers are
# delta arlE1 arlE2 arlE3
# 0.000000 465.0 465.0 465.0
# 0.005623 77.01 85.93 102.68
# 0.007499 64.61 71.78 85.74
# 0.010000 54.20 59.74 71.22
# 0.013335 45.20 49.58 58.90
# 0.017783 37.76 41.06 48.54
# 0.023714 31.54 33.95 39.87
# 0.031623 26.36 28.06 32.68
# 0.042170 22.06 23.19 26.73
# 0.056234 18.49 19.17 21.84
# 0.074989 15.53 15.87 17.83
# 0.100000 13.07 13.16 14.55
# 0.133352 11.03 10.94 11.88
# 0.177828 9.33 9.12 9.71
# 0.237137 7.91 7.62 7.95
# 0.316228 6.72 6.39 6.52
# 0.421697 5.72 5.38 5.37
# 0.562341 4.88 4.54 4.44
# 0.749894 4.18 3.84 3.68
# 1.000000 3.58 3.27 3.07
#
lambda1 <- .133
lambda2 <- .25
lambda3 <- .5
cE1 <- 2.856
cE2 <- 2.974
cE3 <- 3.049
deltas <- 10^(-(18:0)/8)
arlE10 <- round(xewma.arl(lambda1, cE1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10, round(DxDewma.arl(lambda1, cE1, deltas, sided="two"), digits=2))
arlE20 <- round(xewma.arl(lambda2, cE2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20, round(DxDewma.arl(lambda2, cE2, deltas, sided="two"), digits=2))
arlE30 <- round(xewma.arl(lambda3, cE3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30, round(DxDewma.arl(lambda3, cE3, deltas, sided="two"), digits=2))
data.frame(delta=c(0, round(deltas, digits=6)), arlE1, arlE2, arlE3)

## Fahmy/Elsayed (2006) -- two-sided EWMA
## Table 4 (Monte Carlo results, 10^4 replicates, change point at t=51!)
## original numbers are
# delta arl s.e.
# 0.00 365.749 3.598
# 0.10 12.971 0.029
```
```r
lambda <- 0.1
cE <- 2.7
deltas <- c(.1, (1:8)/4)
arlE1 <- c(round(xewma.arl(lambda, cE, 0, sided="two"), digits=3),
            round(DxDewma.arl(lambda, cE, deltas, sided="two"), digits=3))
arlE51 <- c(round(xewma.arl(lambda, cE, 0, sided="two", q=51)[51], digits=3),
            round(DxDewma.arl(lambda, cE, deltas, sided="two", mode="Knoth", q=51),
            digits=3))
data.frame(delta=c(0, deltas), arlE1, arlE51)
```

## additional Monte Carlo results with 10^8 replicates
```r
# delta arl.q=1 s.e. arl.q=51 s.e.
# 0.00 368.910 0.036 361.714 0.038
# 0.10 12.986 0.000 12.781 0.000
# 0.25 7.758 0.000 7.637 0.000
# 0.50 5.318 0.000 5.235 0.000
# 0.75 4.285 0.000 4.218 0.000
# 1.00 3.688 0.000 3.628 0.000
# 1.25 3.274 0.000 3.233 0.000
# 1.50 2.993 0.000 2.942 0.000
# 1.75 2.808 0.000 2.723 0.000
# 2.00 2.616 0.000 2.554 0.000
```

## Zou et al. (2009) -- one-sided EWMA
## Table 1
```r
## original values are
# delta arl1 arl2 arl3
# 0 1730
# 0.0005 317 377 440
# 0.001 215 253 297
# 0.005 83.6 92.6 106
# 0.01 55.6 58.8 66.1
# 0.05 22.6 21.1 22.0
# 0.1 15.5 13.9 13.8
# 0.5 6.65 5.56 5.09
# 1.0 4.67 3.83 3.43
# 2.0 3.21 2.74 2.32
# 3.0 2.86 2.06 1.98
# 4.0 2.14 2.00 1.83
l1 <- 0.03479
l2 <- 0.11125
l3 <- 0.23052
c1 <- 2.711
c2 <- 3.033
```
c3 <- 3.161
zr <- -6
r <- 50
deltas <- c(0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1:4)
arl1 <- c(round(xewma.arl(l1, c1, 0, zr=zr, r=r), digits=2),
        round(DxDewma.arl(l1, c1, deltas, zr=zr, r=r), digits=2))
arl2 <- c(round(xewma.arl(l2, c2, 0, zr=zr), digits=2),
        round(DxDewma.arl(l2, c2, deltas, zr=zr, r=r), digits=2))
arl3 <- c(round(xewma.arl(l3, c3, 0, zr=zr, r=r), digits=2),
        round(DxDewma.arl(l3, c3, deltas, zr=zr, r=r), digits=2))
data.frame(delta=c(0, deltas), arl1, arl2, arl3)
## End(Not run)

xDgrsr.arl

Computes ARLs of Shiryaev-Roberts schemes under drift

**Description**

Computation of the (zero-state and other) Average Run Length (ARL) under drift for Shiryaev-Roberts schemes monitoring normal mean.

**Usage**

```
xDgrsr.arl(k, g, delta, zr = 0, hs = NULL, sided = "one", m = NULL,
mode = "Gan", q = 1, r = 30, with0 = FALSE)
```

**Arguments**

- **k**: reference value of the Shiryaev-Roberts scheme.
- **g**: control limit (alarm threshold) of Shiryaev-Roberts scheme.
- **delta**: true drift parameter.
- **zr**: reflection border for the one-sided chart.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided Shiryaev-Roberts schemes by choosing "one" and "two", respectively. Currently, the two-sided scheme is not implemented.
- **m**: parameter used if mode="Gan". m is design parameter of Gan’s approach. If m=NULL, then m will increased until the resulting ARL does not change anymore.
- **q**: change point position. For q = 1 and μ = μ₀ and μ = μ₁, the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For q > 1 and μ₁ = 0 conditional delays, that is, E_q(L - q + 1|L ≥), will be determined. Note that μ₀=0 is implicitly fixed. Deploy large q to mimic steady-state. It works only for mode="Knoth".
- **mode**: decide whether Gan’s or Knoth’s approach is used. Use "Gan" and "Knoth", respectively. "Knoth" is not implemented yet.
number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).

with0 defines whether the first observation used for the RL calculation follows already 1*delta or still 0*delta. With q additional flexibility is given.

Details

Based on Gan (1991) or Knoth (2003), the ARL is calculated for Shiryaev-Roberts schemes under drift. In case of Gan’s framework, the usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. The framework of Knoth allows to calculate ARLs for varying parameters, such as control limits and distributional parameters. For details see the cited papers.

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References

S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.
C. Zou, Y. Liu and Z. Wang (2009), Comparisons of control schemes for monitoring the means of processes subject to drifts, Metrika 70, 141-163.

See Also

xewma.arl and xewma.ad for zero-state and steady-state ARL computation of EWMA control charts for the classical step change model.

Examples

```r
## Not run:
## Monte Carlo example with 10^8 replicates
# delta  arl  s.e.
# 0.0001 381.8240 0.0304
# 0.0005 238.4630 0.0148
# 0.001 177.4061 0.0097
# 0.002 125.9055 0.0061
# 0.005 75.7574 0.0031
# 0.01 50.2203 0.0018
# 0.02 32.9458 0.0011
# 0.05 18.9213 0.0005
```
# 0.1 12.6054 0.0003
# 0.5 5.2157 0.0001
# 1 3.6537 0.0001
# 3 2.0289 0.0000

k <- .5
L0 <- 500
zr <- -7
r <- 50
g <- xgrsr.crit(k, L0, zr=zr, r=r)
DxDgrsr.arl <- Vectorize(xDgrsr.arl, "delta")
deltas <- c(0.0001, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.5, 1, 3)
arls <- round(DxDgrsr.arl(k, g, deltas, zr=zr, r=r), digits=4)
data.frame(deltas, arls)

## End(Not run)

---

**xDshewhartrunsrules.arl**

*Compute ARLs of Shewhart control charts with and without runs rules under drift*

### Description

Computation of the zero-state Average Run Length (ARL) under drift for Shewhart control charts with and without runs rules monitoring normal mean.

### Usage

```
xDshewhartrunsrules.arl(delta, c = 1, m = NULL, type = "12")
xDshewhartrunsrulesFixedm.arl(delta, c = 1, m = 100, type = "12")
```

### Arguments

- **delta**: true drift parameter.
- **c**: normalizing constant to ensure specific alarming behavior.
- **type**: controls the type of Shewhart chart used, seed details section.
- **m**: parameter of Gan’s approach. If m=NULL, then m will increased until the resulting ARL does not change anymore.

### Details

Based on Gan (1991), the ARL is calculated for Shewhart control charts with and without runs rules under drift. The usual ARL function with mu=m*delta is determined and recursively via m-1, m-2, ... 1 (or 0) the drift ARL determined. `xDshewhartrunsrulesFixedm.arl` is the actual work horse, while `xDshewhartrunsrules.arl` provides a convenience wrapper. Note that Aerne et al. (1991) deployed a method that is quite similar to Gan’s algorithm. For type see the help page of `xshewhartrunsrules.arl`. 
Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


See Also

xshewhartrunsrules.arl for zero-state ARL computation of Shewhart control charts with and without runs rules for the classical step change model.

Examples

```r
## Aerne et al. (1991)
## Table I (continued)
## original numbers are
# delta arl1of1 arl2of3 arl4of5 arl10
# 0.005623 136.67 120.90 105.34 107.08
# 0.007499 114.98 101.23 88.09 89.94
# 0.010000 96.03 84.22 73.31 75.23
# 0.013335 79.69 69.68 60.75 62.73
# 0.017783 65.75 57.38 50.18 52.18
# 0.023714 53.99 47.06 41.33 43.35
# 0.031623 44.15 38.47 33.99 36.00
# 0.042170 35.97 31.36 27.91 29.90
# 0.056234 29.21 25.51 22.91 24.86
# 0.074989 23.65 20.71 18.81 20.70
# 0.100000 19.11 16.79 15.45 17.29
# 0.133352 15.41 13.61 12.72 14.47
# 0.177828 12.41 11.03 10.50 12.14
# 0.237137 9.98 8.94 8.71 10.18
# 0.316228 8.02 7.25 7.26 8.45
# 0.421697 6.44 5.89 6.09 6.84
# 0.562341 5.17 4.80 5.15 5.48
# 0.749894 4.16 3.92 4.36 4.39
# 1.000000 3.35 3.22 3.63 3.52
c1of1 <- 3.069/3
c2of3 <- 2.1494/2
c4of5 <- 1.14
c10 <- 3.2425/3
DxDshewhartrunsrules.arl <- Vectorize(xDshewhartrunsrules.arl, "delta")
deltas <- 10^(-(18:0)/8)
arl1of1 <- round(DxDshewhartrunsrules.arl(deltas, c=c1of1, type="1"), digits=2)
```
arl2of3 <-
round(DxDshewhartrunsrules.arl(deltas, c=c2of3, type="12"), digits=2)
arl4of5 <-
round(DxDshewhartrunsrules.arl(deltas, c=c4of5, type="13"), digits=2)
arl10 <-
round(DxDshewhartrunsrules.arl(deltas, c=c10, type="SameSide10"), digits=2)
data.frame(delta=round(deltas, digits=6), arl1of1, arl2of3, arl4of5, arl10)

---

**xewma.ad**

*Compute steady-state ARLs of EWMA control charts*

**Description**

Computation of the steady-state Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean.

**Usage**

```r
xewma.ad(l, c, mu1, mu0=0, zr=0, z0=0, sided="one", limits="fix",
steady.state.mode="conditional", r=40)
```

**Arguments**

- **l**: smoothing parameter lambda of the EWMA control chart.
- **c**: critical value (similar to alarm limit) of the EWMA control chart.
- **mu1**: out-of-control mean.
- **mu0**: in-control mean.
- **zr**: reflection border for the one-sided chart.
- **z0**: restarting value of the EWMA sequence in case of a false alarm in `steady.state.mode="cyclical"`.
- **sided**: distinguishes between one- and two-sided two-sided EWMA control chart by choosing "one" and "two", respectively.
- **limits**: distinguishes between different control limits behavior.
- **steady.state.mode**: distinguishes between two steady-state modes – conditional and cyclical.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1` (one-sided) or `r` (two-sided).

**Details**

*xewma.ad* determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nyström method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.
Value

Returns a single value which resembles the steady-state ARL.

Author(s)

Sven Knoth

References


See Also

`xewma.arl` for zero-state ARL computation and `xcusum.ad` for the steady-state ARL of CUSUM control charts.

Examples

```r
## comparison of zero-state (= worst case) and steady-state performance
## for two-sided EWMA control charts

l <- .1

mu <- c(0, .5, 1, 1.5, 2)

l <- .1

c <- xewma.crit(l, 500, sided = "two")

arl <- sapply(mu, l = l, c = c, sided = "two", xewma.arl)

ad <- sapply(mu, l = l, c = c, sided = "two", xewma.ad)

round(cbind(mu, arl, ad), digits = 2)

## Lucas/Saccucci (1990)
## two-sided EWMA

l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437

mu <- c(0, .25, .5, .75, 1, 1.5, 2, 2.5, 3, 3.5, 4, 5)

ad1 <- sapply(mu, l = l1, c = c1, sided = "two", xewma.ad)

ad2 <- sapply(mu, l = l2, c = c2, sided = "two", xewma.ad)

round(cbind(mu, ad1, ad2), digits = 2)

## original results are (in Table 3)
## 0.00 499. 480.
## 0.25 254. 74.1
## 0.50 88.4 28.6
```
xewma.arl

## Compute ARLs of EWMA control charts

### Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean.

### Usage

```r
xewma.arl(l, c, mu, zr = 0, hs = 0, sided = "one", limits = "fix", q = 1,
steady.state.mode = "conditional", r = 40)
```

### Arguments

- `l`: smoothing parameter lambda of the EWMA control chart.
- `c`: critical value (similar to alarm limit) of the EWMA control chart.
- `mu`: true mean.
- `zr`: reflection border for the one-sided chart.
- `hs`: so-called headstart (enables fast initial response).
- `sided`: distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
- `limits`: distinguishes between different control limits behavior.
- `q`: change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1|L \geq q) \), will be determined. Note that \( \mu_0=0 \) is implicitly fixed.
- `steady.state.mode`: distinguishes between two steady-state modes – conditional and cyclical (needed for \( q > 1 \)).
- `r`: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-sided) or \( r \) (two-sided).
Details

In case of the EWMA chart with fixed control limits, `xewma.arl` determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature. If limits is not "fix", then the method presented in Knoth (2003) is utilized. Note that for one-sided EWMA charts (sided="one"), only "vacl" and "stat" are deployed, while for two-sided ones (sided="two") also "fir", "both" (combination of "fir" and "vacl"), and "Steiner" are implemented. For details see Knoth (2004).

Value

Except for the fixed limits EWMA charts it returns a single value which resembles the ARL. For fixed limits charts, it returns a vector of length q which resembles the ARL and the sequence of conditional expected delays for q=1 and q>1, respectively.

Author(s)

Sven Knoth

References


See Also

`xcusum.arl` for zero-state ARL computation of CUSUM control charts and `xewma.ad` for the steady-state ARL.

Examples

```R
## Waldmann (1986), one-sided EWMA
l <- .75
round(xewma.arl(l,2*sqrt((2-l)/l),0,zr=-4*sqrt((2-l)/l)),digits=1)
```
l <- .5
round(xewma.arl(l,2*sqrt((2-l)/l),0,zr=-4*sqrt((2-l)/l)),digits=1)
## original values are 209.3 and 3907.5 (in Table 2).

## Waldmann (1986), two-sided EWMA with fixed control limits
l <- .75
round(xewma.arl(l,2*sqrt((2-l)/l),0,sided="two"),digits=1)
l <- .5
round(xewma.arl(l,2*sqrt((2-l)/l),0,sided="two"),digits=1)
## original values are 104.0 and 1952 (in Table 1).

## Crowder (1987), two-sided EWMA with fixed control limits
l1 <- .5
l2 <- .05
c <- 2
mu <- (0:16)/4
arl1 <- sapply(mu,l=l1,c=c,sided="two",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c,sided="two",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)
## original results are (in Table 1)
## 0.00 26.45 127.53
## 0.25 20.12 43.94
## 0.50 11.89 18.97
## 0.75 7.29 11.64
## 1.00 4.91 8.38
## 1.25 3.95* 6.56
## 1.50 2.80 5.41
## 1.75 2.29 4.62
## 2.00 1.94 4.04
## 2.25 1.70 3.61
## 2.50 1.51 3.26
## 2.75 1.37 2.99
## 3.00 1.26 2.76
## 3.25 1.18 2.56
## 3.50 1.12 2.39
## 3.75 1.08 2.26
## 4.00 1.05 2.15 (* -- in Crowder (1987) typo!?)

## Lucas/Saccucci (1990)
## two-sided EWMA

## with fixed limits
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
mu <- c(0,.25,.5,.75,1,1.5,2,2.5,3,3.5,4,5)
arl1 <- sapply(mu,l=l1,c=c1,sided="two",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c2,sided="two",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)
## original results are (in Table 3)
### 0.00 500. 500.
### 0.25 255. 76.7
### 0.50 88.8 29.3
### 0.75 35.9 17.6
### 1.00 17.5 12.6
### 1.50 6.53 8.07
### 2.00 3.63 5.99
### 2.50 2.50 4.80
### 3.00 1.93 4.03
### 3.50 1.58 3.49
### 4.00 1.34 3.11
### 5.00 1.07 2.55

## Not run:
## with fir feature
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
hs1 <- c1/2
hs2 <- c2/2
mu <- c(0,.5,1,2,3,5)
arl1 <- sapply(mu,l=l1,c=c1,hs=hs1,sided="two",limits="fir",xewma.arl)
arl2 <- sapply(mu,l=l2,c=c2,hs=hs2,sided="two",limits="fir",xewma.arl)
round(cbind(mu,arl1,arl2),digits=2)

## original results are (in Table 5)
## 0.0 487. 406.
## 0.5 86.1 18.4
## 1.0 15.9 7.36
## 2.0 2.87 3.43
## 3.0 1.45 2.34
## 5.0 1.01 1.57

## Chandrasekaran, English, Disney (1995)
## two-sided EWMA with fixed and variance adjusted limits (vacl)
l1 <- .25
l2 <- .1
c1s <- 2.9985
c1n <- 3.0042
c2s <- 2.8159
c2n <- 2.8452
mu <- c(0,.25,.5,.75,1,2)
arl1s <- sapply(mu,l=l1,c=c1s,sided="two",xewma.arl)
arl1n <- sapply(mu,l=l1,c=c1n,sided="two",limits="vacl",xewma.arl)
arl2s <- sapply(mu,l=l2,c=c2s,sided="two",xewma.arl)
arl2n <- sapply(mu,l=l2,c=c2n,sided="two",limits="vacl",xewma.arl)
round(cbind(mu,arl1s,arl1n,arl2s,arl2n),digits=2)

## original results are (in Table 2)
## 0.00 500. 500. 500. 500.
## 0.25 170.09 167.54 105.90 96.6
## 0.50 48.14 45.65 31.08 24.35
## 0.75 20.02 19.72 15.71 10.74
## 1.00 11.07 9.37 10.23 6.35
## 2.00 3.59 2.64 4.32 2.73

The results in Chandrasekaran, English, Disney (1995) are not that accurate. Let us consider the more appropriate comparison

```r
c1s <- xewma.crit(l1,500,sided="two")
c1n <- xewma.crit(l1,500,sided="two",limits="vacl")
c2s <- xewma.crit(l2,500,sided="two")
c2n <- xewma.crit(l2,500,sided="two",limits="vacl")
mu <- c(0,.25,.5,.75,1,2)
arl1s <- sapply(mu,l=l1,c=c1s,sided="two",xewma.arl)
arl1n <- sapply(mu,l=l1,c=c1n,sided="two",limits="vacl",xewma.arl)
arl2s <- sapply(mu,l=l2,c=c2s,sided="two",xewma.arl)
arl2n <- sapply(mu,l=l2,c=c2n,sided="two",limits="vacl",xewma.arl)
round(cbind(mu,arl1s,arl1n,arl2s,arl2n),digits=2)
```

## which demonstrate the abilities of the variance-adjusted limits scheme more explicitly.

## Rhoads, Montgomery, Mastrangelo (1996)
## two-sided EWMA with fixed and variance adjusted limits (vacl),
## with fir and both features

```r
l <- .03
c <- 2.437
mu <- c(0,.5,1,1.5,2,3,4)
sl <- sqrt(l*(2-l))
arlfix <- sapply(mu,l=l,c=c,sided="two",xewma.arl)
arlvacl <- sapply(mu,l=l,c=c,sided="two",limits="vacl",xewma.arl)
arlfir <- sapply(mu,l=l,c=c,hs=c/2,sided="two",limits="fir",xewma.arl)
arlboth <- sapply(mu,l=l,c=c,hs=c/2*sl,sided="two",limits="both",xewma.arl)
round(cbind(mu,arlfix,arlvacl,arlfir,arlboth),digits=1)
```

## original results are (in Table 1)
## 0.0 477.3* 427.9* 383.4* 286.2*
## 0.5 29.7 20.0 18.6 12.8
## 1.0 12.5 6.5 7.4 3.6
## 1.5 8.1 3.3 4.6 1.9
## 2.0 6.0 2.2 3.4 1.4
## 3.0 4.0 1.3 2.4 1.0
## 4.0 3.1 1.1 1.9 1.0
## * -- the in-control values differ sustainably from the true values!

## Steiner (1999)
## two-sided EWMA control charts with various modifications

## fixed vs. variance adjusted limits

```r
l <- .05
c <- 3
mu <- c(0,.25,.5,1,1.5,2,2.5,3,3.5,4)
arlfix <- sapply(mu,l=1,c=c, sided="two", xewma.arl)
arlvac1 <- sapply(mu,l=1,c=c, sided="two", limits="vacl", xewma.arl)
round(cbind(mu, arlfix, arlvac1), digits=1)

## original results are (in Table 2)
## 0.00 1379.0 1353.0
## 0.25 135.0 127.0
## 0.50 37.4 32.5
## 0.75 20.0 15.6
## 1.00 13.5 9.0
## 1.50 8.3 4.5
## 2.00 6.0 2.8
## 2.50 4.8 2.0
## 3.00 4.0 1.6
## 3.50 3.4 1.3
## 4.00 3.0 1.1

## fir, both, and Steiner's modification
l <- .03
cfir <- 2.44
cboth <- 2.54
cstein <- 2.55
hsfir <- cfir/2
hsboth <- cboth/2*sqrt(l*(2-l))
mu <- c(0,.5,1,1.5,2,3,4)
arlfir <- sapply(mu,l=1,c=cfir,hs=hsfir, sides="two", limits="fir", xewma.arl)
arlboth <- sapply(mu,l=1,c=cboth,hs=hsboth, sides="two", limits="both", xewma.arl)
arlstein <- sapply(mu,l=1,c=cstein, sides="two", limits="Steiner", xewma.arl)
round(cbind(mu, arlfir, arlboth, arlstein), digits=1)

## original values are (in Table 5)
## 0.0 383.0 384.0 391.0
## 0.5 18.6 14.9 13.8
## 1.0 7.4 3.9 3.6
## 1.5 4.6 2.0 1.8
## 2.0 3.4 1.4 1.3
## 3.0 2.4 1.1 1.0
## 4.0 1.9 1.0 1.0

## SAS/QC manual 1999
## two-sided EWMA control charts with fixed limits
l <- .25
c <- 3
mu <- 1
print(xewma.arl(l,c,mu, sides="two"), digits=11)

# original value is 11.154267016.

## Some recent examples for one-sided EWMA charts
## with varying limits and in the so-called stationary mode
## xewma.arl.f

### Description

Computation of the (zero-state) Average Run Length (ARL) function for different types of EWMA control charts monitoring normal mean.
Usage

xewma.arl.f(l,c,\mu,zr=0,sided="one",limits="fix",r=40)

Arguments

l \quad \text{smoothing parameter lambda of the EWMA control chart.}
c \quad \text{critical value (similar to alarm limit) of the EWMA control chart.}
\mu \quad \text{true mean.}
zr \quad \text{reflection border for the one-sided chart.}
sided \quad \text{distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.}
limits \quad \text{distinguishes between different control limits behavior.}
r \quad \text{number of quadrature nodes, dimension of the resulting linear equation system is equal to } r+1 \text{ (one-sided) or } r \text{ (two-sided).}

Details

It is a convenience function to yield the ARL as function of the head start hs. For more details see xewma.arl.

Value

It returns a function of a single argument, hs=x which maps the head-start value hs to the ARL.

Author(s)

Sven Knoth

References


See Also

xewma.arl for zero-state ARL for one specific head-start hs.

Examples

# will follow
**xewma.arl.prerun**

*Compute ARLs of EWMA control charts in case of estimated parameters*

---

**Description**

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.

**Usage**

```r
xewma.arl.prerun(l, c, mu, zr=0, hs=0, sided="two", limits="fix", q=1, size=100, df==NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10)
```

```r
xewma.crit.prerun(l, L0, mu, zr=0, hs=0, sided="two", limits="fix", size=100, df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, c.error=1e-12, L.error=1e-9, OUTPUT=FALSE)
```

**Arguments**

- `l`: smoothing parameter lambda of the EWMA control chart.
- `c`: critical value (similar to alarm limit) of the EWMA control chart.
- `mu`: true mean shift.
- `zr`: reflection border for the one-sided chart.
- `hs`: so-called headstart (give fast initial response).
- `sided`: distinguish between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
- `limits`: distinguish between different control limits behavior.
- `q`: change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu_i = 0 \) conditional delays, that is, \( E_q(L - q + 1 | L \geq q) \), will be determined. Note that \( \mu_0=0 \) is implicitly fixed.
- `size`: pre run sample size.
- `df`: Degrees of freedom of the pre run variance estimator. Typically it is simply size - 1. If the pre run is collected in batches, then also other values are needed.
- `estimated`: name the parameter to be estimated within the "mu", "sigma", "both".
- `qm.mu`: number of quadrature nodes for convoluting the mean uncertainty.
- `qm.sigma`: number of quadrature nodes for convoluting the standard deviation uncertainty.
- `truncate`: size of truncated tail.
- `L0`: in-control ARL.
- `c.error`: error bound for two succeeding values of the critical value during applying the secant rule.
- `L.error`: error bound for the ARL level \( L_0 \) during applying the secant rule.
- `OUTPUT`: activate or deactivate additional output.
Details

Essentially, the ARL function `xewma.arl` is convoluted with the distribution of the sample mean, standard deviation or both. For details see Jones/Champ/Rigdon (2001) and Knoth (2014?).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


S. Knoth (2014?), tbd, tbd, tbd-tbd.

See Also

`xewma.arl` for the usual zero-state ARL computation.

Examples

```r
## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )

n <- 5 # sample size
m <- 20 # pre run with 20 samples of size n = 5
C4m <- c4m(m, n) # needed for bias correction

# Table 1, 3rd column
lambda <- 0.2
L <- 2.636
xewma.ARL <- Vectorize("xewma.arl", "mu")
xewma.ARL.prerun <- Vectorize("xewma.arl.prerun", "mu")

mu <- c(0, .25, .5, 1, 1.5, 2)
ARL <- round(xewma.ARL(lambda, L, mu, sided="two"), digits=2)
p.ARL <- round(xewma.ARL.prerun(lambda, L/C4m, mu, sided="two", size=m, df=m*(n-1), estimated="both", qm.mu=70), digits=2)

# Monte-Carlo with 10^8 repetitions: 200.325 (0.020) and 144.458 (0.022)
cbind(mu, ARL, p.ARL)
```
## Not run:

# Figure 5, subfigure r = 0.2
mu_ <- (seq(0:85)/40)
ARL_ <- round(xewma.ARL(lambda, L, mu_, sided="two"), digits=2)
p.ARL_ <- round(xewma.ARL.prerun(lambda, L/C4m, mu_, sided="two", size=m, df=m*(n-1), estimated="both"), digits=2)

plot(mu_, ARL_, type="l", xlab=expression(delta), ylab="ARL", xlim=c(0,2))
abline(v=0, h=0, col="grey", lwd=.7)
points(mu, ARL, pch=5)
lines(mu, p.ARL_, col="blue")
points(mu, p.ARL, pch=18, col ="blue")
legend("topright", c("Known", "Estimated"), col=c("black", "blue"), lty=1, pch=c(5, 18))

## End(Not run)

---

**xewma.crit**

*Compute critical values of EWMA control charts*

**Description**

Computation of the critical values (similar to alarm limits) for different types of EWMA control charts monitoring normal mean.

**Usage**

```r
xewma.crit(l, L0, mu0=0, zr=0, hs=0, sided="one", limits="fix", r=40, c0=NULL)
```

**Arguments**

- `l`: smoothing parameter lambda of the EWMA control chart.
- `L0`: in-control ARL.
- `mu0`: in-control mean.
- `zr`: reflection border for the one-sided chart.
- `hs`: so-called headstart (enables fast initial response).
- `sided`: distinguishes between one- and two-sided two-sided EWMA control chart by choosing "one" and "two", respectively.
- `limits`: distinguishes between different control limits behavior.
- `r`: number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).
- `c0`: starting value for iteration rule.

**Details**

`xewma.crit` determines the critical values (similar to alarm limits) for given in-control ARL `L0` by applying secant rule and using `xewma.arl()`. 
**Value**

Returns a single value which resembles the critical value \( c \).

**Author(s)**

Sven Knoth

**References**


**See Also**

xewma.arl for zero-state ARL computation.

**Examples**

```r
l <- .1
incontrolARL <- c(500,5000,50000)
sapply(incontrolARL,l=l,sided="two",xewma.crit,r=35) # accuracy with 35 nodes
sapply(incontrolARL,l=l,sided="two",xewma.crit) # accuracy with 40 nodes
sapply(incontrolARL,l=l,sided="two",xewma.crit,r=50) # accuracy with 50 nodes

## Crowder (1989)
## two-sided EWMA control charts with fixed limits
l <- c(.05,.1,.15,.2,.25)
L0 <- 250
round(sapply(l,L0=L0,sided="two",xewma.crit),digits=2)
## original values are 2.32, 2.55, 2.65, 2.72, and 2.76.
```

---

**xewma.q**

*Compute RL quantiles of EWMA control charts*

**Description**

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean.

**Usage**

```r
xewma.q(l, c, mu, alpha, zr=0, hs=0, sided="two", limits="fix", q=1, r=40)

xewma.q.crit(l, L0, mu, alpha, zr=0, hs=0, sided="two", limits="fix", r=40,
c.error=1e-12, a.error=1e-9, OUTPUT=FALSE)
```
Arguments

l  smoothing parameter lambda of the EWMA control chart.

c  critical value (similar to alarm limit) of the EWMA control chart.

mu  true mean.

alpha  quantile level.

zr  reflection border for the one-sided chart.

hs  so-called headstart (enables fast initial response).

sided  distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.

limits  distinguishes between different control limits behavior.

q  change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1|L \geq \cdot) \), will be determined. Note that \( \mu_0=0 \) is implicitly fixed.

r  number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-sided) or \( r \) (two-sided).

L0  in-control quantile value.

c.error  error bound for two succeeding values of the critical value during applying the secant rule.

a.error  error bound for the quantile level alpha during applying the secant rule.

OUTPUT  activate or deactivate additional output.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. If \texttt{limits} is not "fix", then the method presented in Knoth (2003) is utilized. Note that for one-sided EWMA charts (\texttt{sided="one"}), only "vacl" and "stat" are deployed, while for two-sided ones (\texttt{sided="two"}) also "fir", "both" (combination of "fir" and "vacl"), and "Steiner" are implemented. For details see Knoth (2004).

Value

Returns a single value which resembles the RL quantile of order \( q \).

Author(s)

Sven Knoth

References


S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.


**See Also**

*xewma.arl* for zero-state ARL computation of EWMA control charts.

**Examples**

```r
## Gan (1993), two-sided EWMA with fixed control limits
## some values of his Table 1 -- any median RL should be 500
XEWMA.Q <- Vectorize("xewma.q", c("l", "c"))
G.lambda <- c(.05, .1, .15, .2, .25)
G.h <- c(.441, .675, .863, 1.027, 1.177)
MEDIAN <- ceiling(XEWMA.Q(G.lambda, G.h/sqrt(G.lambda/(2-G.lambda)),
0, .5, sided="two"))
print(cbind(G.lambda, MEDIAN))

## increase accuracy of thresholds
# (i) calculate threshold for given in-control median value by
# deploying secant rule
XEWMA.q.crit <- Vectorize("xewma.q.crit", "l")

# (ii) re-calculate the thresholds and remove the standardization step
L0 <- 500
G.h.new <- XEWMA.q.crit(G.lambda, L0, 0, .5, sided="two")
G.h.new <- round(G.h.new * sqrt(G.lambda/(2-G.lambda)), digits=5)

# (iii) compare Gan’s original values and the new ones with 5 digits
print(cbind(G.lambda, G.h.new, G.h))

# (iv) calculate the new medians
MEDIAN <- ceiling(XEWMA.Q(G.lambda, G.h.new/sqrt(G.lambda/(2-G.lambda)),
0, .5, sided="two"))
print(cbind(G.lambda, MEDIAN))
```

**xewma.q.prerun**  
Compute RL quantiles of EWMA control charts in case of estimated parameters

**Description**

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.
Usage

xewma.q.prerun(l, c, mu, p, zr=0, hs=0, sided="two", limits="fix", q=1, size=100,
               df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, bound=1e-10)

xewma.q.crit.prerun(l, L0, mu, p, zr=0, hs=0, sided="two", limits="fix", size=100,
              df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, bound=1e-10,
              c.error=1e-10, p.error=1e-9, OUTPUT=FALSE)

Arguments

l        smoothing parameter lambda of the EWMA control chart.
c        critical value (similar to alarm limit) of the EWMA control chart.
mu       true mean shift.
p        quantile level.
zr       reflection border for the one-sided chart.
hs       so-called headstart (give fast initial response).
sided    distinguish between one- and two-sided EWMA control chart by choosing "one"
          and "two", respectively.
limits    distinguish between different control limits behavior.
q         change point position. For q = 1 and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state
          ARLs for the in-control and out-of-control case, respectively, are calculated.
          For q > 1 and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1|L \geq) \), will be
          determined. Note that \( mu=0 \) is implicitly fixed.
size     pre run sample size.
df       Degrees of freedom of the pre run variance estimator. Typically it is simply size
          - 1. If the pre run is collected in batches, then also other values are needed.
estimated name the parameter to be estimated within the "mu", "sigma", "both".
qm.mu     number of quadrature nodes for convoluting the mean uncertainty.
qm.sigma  number of quadrature nodes for convoluting the standard deviation uncertainty.
truncate  size of truncated tail.
bound    control when the geometric tail kicks in; the larger the quicker and less accurate;
          bound should be larger than 0 and less than 0.001.
L0        in-control quantile value.
c.error   error bound for two succeeding values of the critical value during applying the
          secant rule.
p.error   error bound for the quantile level p during applying the secant rule.
OUTPUT    activate or deactivate additional output.

Details

Essentially, the ARL function xewma.q is convoluted with the distribution of the sample mean,
standard deviation or both. For details see Jones/Champ/Rigdon (2001) and Knoth (2014?).
Value

Returns a single value which resembles the RL quantile of order q.

Author(s)

Sven Knoth

References


S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, Statistical Papers 46, 47-64.

S. Knoth (2014?), tbd, tbd, tbd-tbd.


See Also

xewma.q for the usual RL quantiles computation of EWMA control charts.

Examples

```r
## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )

n <- 5 # sample size
m <- 20 # pre run with 20 samples of size n = 5
C4m <- c4m(m, n) # needed for bias correction

# Table 1, 3rd column
lambda <- 0.2
L <- 2.636

xewma.Q <- Vectorize("xewma.q", "mu")
xewma.Q.prerun <- Vectorize("xewma.q.prerun", "mu")

mu <- c(0, .25, .5, 1, 1.5, 2)
Q1 <- ceiling(xewma.Q(lambda, L, mu, 0.1, sided="two"))
Q2 <- ceiling(xewma.Q(lambda, L, mu, 0.5, sided="two"))
Q3 <- ceiling(xewma.Q(lambda, L, mu, 0.9, sided="two"))

cbind(mu, Q1, Q2, Q3)

## Not run:
p.Q1 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.1, sided="two",
size=m, df=m*(n-1), estimated="both")
```
p.Q2 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.5, sided="two", size=m, df=m*(n-1), estimated="both")
p.Q3 <- xewma.Q.prerun(lambda, L/C4m, mu, 0.9, sided="two", size=m, df=m*(n-1), estimated="both")
cbind(mu, p.Q1, p.Q2, p.Q3)

## End(Not run)

## original values are
#     mu  Q1  Q2  Q3  p.Q1  p.Q2  p.Q3
# 0.00  25 140 456  13   73  345
# 0.25  12  56 174   9   46  253
# 0.50   7 20  56   6  20  101
# 1.00   4  7  15   3   7  18
# 1.50   3  4  7   2   4   8
# 2.00   2  3  5   2   3   5

xewma.sf

Compute the survival function of EWMA run length

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

xewma.sf(l, c, mu, n, zr=0, hs=0, sided="one", limits="fix", q=1, r=40)

Arguments

l
  smoothing parameter lambda of the EWMA control chart.

c
  critical value (similar to alarm limit) of the EWMA control chart.

mu
  true mean.

n
  calculate sf up to value n.

zr
  reflection border for the one-sided chart.

hs
  so-called headstart (enables fast initial response).

sided
  distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.

limits
  distinguishes between different control limits behavior.

q
  change point position. For q = 1 and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that mu0=0 is implicitely fixed.

r
  number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).
Details

The survival function \( P(L>n) \) and derived from it also the cdf \( P(L<=n) \) and the pmf \( P(L=n) \) illustrate the distribution of the EWMA run length. For large \( n \) the geometric tail could be exploited. That is, with reasonable large \( n \) the complete distribution is characterized. The algorithm is based on Waldmann's survival function iteration procedure. For varying limits and for change points after 1 the algorithm from Knoth (2004) is applied. Note that for one-sided EWMA charts (sided="one"), only "vacl" and "stat" are deployed, while for two-sided ones (sided="two") also "fir", "both" (combination of "fir" and "vacl"), and "Steiner" are implemented. For details see Knoth (2004).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References


S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, Statistical Papers 46, 47-64.


See Also

xewma.arl for zero-state ARL computation of EWMA control charts.

Examples

```r
## Gan (1993), two-sided EWMA with fixed control limits
## some values of his Table 1 -- any median RL should be 500

G.lambda <- c(.05, .1, .15, .2, .25)
G.h <- c(.441, .675, .863, 1.027, 1.177)/sqrt(G.lambda/(2-G.lambda))

for ( i in 1:length(G.lambda) ) {
  SF <- xewma.sf(G.lambda[i], G.h[i], 0, 1000)
  if (i==1) plot(1:length(SF), SF, type="l", xlab="n", ylab="P(L>n)"
  else lines(1:length(SF), SF, col=i)
}
Compute the survival function of EWMA run length in case of estimated parameters

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean if the in-control mean, standard deviation, or both are estimated by a pre run.

Usage

```r
xewma.sf.prerun(l, c, mu, n, zr=0, hs=0, sided="one", limits="fix", q=1, size=100, df=NULL, estimated="mu", qm.mu=30, qm.sigma=30, truncate=1e-10, tail_approx=TRUE, bound=1e-10)
```

Arguments

- **l**: smoothing parameter lambda of the EWMA control chart.
- **c**: critical value (similar to alarm limit) of the EWMA control chart.
- **mu**: true mean.
- **n**: calculate sf up to value `n`.
- **zr**: reflection border for the one-sided chart.
- **hs**: so-called headstart (give fast initial response).
- **sided**: distinguish between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
- **limits**: distinguish between different control limits behavior.
- **q**: change point position. For `q = 1` and `μ = μ₀` and `μ = μ₁`, the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that `mu0=0` is implicitly fixed.
- **size**: pre run sample size.
- **df**: degrees of freedom of the pre run variance estimator. Typically it is simply `size` - 1. If the pre run is collected in batches, then also other values are needed.
- **estimated**: name the parameter to be estimated within the "mu", "sigma", "both".
- **qm.mu**: number of quadrature nodes for convoluting the mean uncertainty.
- **qm.sigma**: number of quadrature nodes for convoluting the standard deviation uncertainty.
- **truncate**: size of truncated tail.
- **tail_approx**: Controls whether the geometric tail approximation is used (is faster) or not.
- **bound**: control when the geometric tail kicks in; the larger the quicker and less accurate; `bound` should be larger than 0 and less than 0.001.

Details

The survival function `P(L>n)` and derived from it also the cdf `P(L<=n)` and the pmf `P(L=n)` illustrate the distribution of the EWMA run length...
xewma.sf.prerun

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References


S. Knoth (2003), EWMA schemes with non-homogeneous transition kernels, Sequential Analysis 22, 241-255.

S. Knoth (2004), Fast initial response features for EWMA Control Charts, Statistical Papers 46, 47-64.


See Also

xewma.sf for the RL survival function of EWMA control charts w/o pre run uncertainty.

Examples

## Jones/Champ/Rigdon (2001)

c4m <- function(m, n) sqrt(2)*gamma( (m*(n-1)+1)/2 )/sqrt( m*(n-1) )/gamma( m*(n-1)/2 )
n <- 5 # sample size

# Figure 6, subfigure r=0.1
lambda <- 0.1
L <- 2.454

CDF0 <- 1 - xewma.sf(lambda, L, 0, 600, sided="two")
m <- 10 # pre run size

CDF1 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two", size=m, df=m*(n-1), estimated="both")
m <- 20

CDF2 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two", size=m, df=m*(n-1), estimated="both")
m <- 50

CDF3 <- 1 - xewma.sf.prerun(lambda, L/c4m(m,n), 0, 600, sided="two", size=m, df=m*(n-1), estimated="both")

plot(CDF0, type="l", xlab="t", ylab=expression(P(T<=t)), xlim=c(0,500), ylim=c(0,1))

abline(v=0, h=c(0,1), col="grey", lwd=.7)
xgrsr.ad

**Compute steady-state ARLs of Shiryaev-Roberts schemes**

**Description**

Computation of the steady-state Average Run Length (ARL) for Shiryaev-Roberts schemes monitoring normal mean.

**Usage**

```r
xgrsr.ad(k, g, mu1, mu0 = 0, zr = 0, sided = "one", MPT = FALSE, r = 30)
```

**Arguments**

- `k`: reference value of the Shiryaev-Roberts scheme.
- `g`: control limit (alarm threshold) of Shiryaev-Roberts scheme.
- `mu1`: out-of-control mean.
- `mu0`: in-control mean.
- `zr`: reflection border to enable the numerical algorithms used here.
- `sided`: distinguishes between one- and two-sided schemes by choosing "one" and "two", respectively. Currently only one-sided schemes are implemented.
- `MPT`: switch between the old implementation (FALSE) and the new one (TRUE) that considers the completed likelihood ratio. MPT contains the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
- `r`: number of quadrature nodes, dimension of the resulting linear equation system is equal to `r+1`.

**Details**

`xgrsr.ad` determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

**Value**

Returns a single value which resembles the steady-state ARL.
Author(s)

Sven Knoth

References


See Also

xewma.arl and xcusum.arl for zero-state ARL computation of EWMA and CUSUM control charts, respectively, and xgrsr.arl for the zero-state ARL.

Examples

## Small study to identify appropriate reflection border to mimic unreflected schemes
k <- .5
g <- log(390)
zrs <- -(0:10)
ZRxgrsr.ad <- Vectorize(xgrsr.ad, "zr")
ads <- ZRxgrsr.ad(k, g, 0, zr=zrs)
data.frame(zrs, ads)

## Table 2 from Knoth (2006)
## original values are
# mu arl
# 0 689
# 0.5 30
# 1 8.9
# 1.5 5.1
# 2 3.6
# 2.5 2.8
# 3 2.4
#
# k <- .5
g <- log(390)
zs <- -5 # see first example
mus <- (0:6)/2
Mxgrsr.ad <- Vectorize(xgrsr.ad, "mu1")
ads <- round(Mxgrsr.ad(k, g, mus, zr=zs), digits=1)
data.frame(mus, ads)

## Table 4 from Moustakides et al. (2009)
## original values are
## gamma A STADD/steady-state ARL
# 50 28.02 4.37
# 100 56.04 5.46
# 500 280.19  8.33  
# 1000 560.37  9.64  
# 5000 2801.75 12.79  
# 10000 5603.7 14.17  
Gxgrsr.ad <- Vectorize("xgrsr.ad", "g")
As <- c(28.02, 56.04, 280.19, 560.37, 2801.75, 5603.7)
gs <- log(As)
theta <- 1
zr <- -6
ads <- round(Gxgrsr.ad(theta/2, gs, theta, zr=zr, r=100), digits=2)
data.frame(As, ads)

---

**xgrsr.arl**  
*Compute (zero-state) ARLs of Shiryaev-Roberts schemes*

**Description**

Computation of the (zero-state) Average Run Length (ARL) for Shiryaev-Roberts schemes monitoring normal mean.

**Usage**

xgrsr.arl(k, g, mu, zr = 0, hs=NULL, sided = "one", q = 1, MPT = FALSE, r = 30)

**Arguments**

- **k**: reference value of the Shiryaev-Roberts scheme.
- **g**: control limit (alarm threshold) of Shiryaev-Roberts scheme.
- **mu**: true mean.
- **zr**: reflection border to enable the numerical algorithms used here.
- **hs**: so-called headstart (enables fast initial response). If hs=NULL, then the classical headstart -Inf is used (corresponds to 0 for the non-log scheme).
- **sided**: distinguishes between one- and two-sided schemes by choosing "one" and "two", respectively. Currently only one-sided schemes are implemented.
- **q**: change point position. For q = 1 and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1 | L \geq q) \), will be determined. Note that \( mu0=0 \) is implicitly fixed.
- **MPT**: switch between the old implementation (FALSE) and the new one (TRUE) that considers the complete likelihood ratio. MPT stands for the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \).
Details

xgrsr.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

Value

Returns a vector of length q which resembles the ARL and the sequence of conditional expected delays for q=1 and q>1, respectively.

Author(s)

Sven Knoth

References


See Also

xewma.arl and xcusum-arl for zero-state ARL computation of EWMA and CUSUM control charts, respectively, and xgrsr.ad for the steady-state ARL.

Examples

## Small study to identify appropriate reflection border to mimic unreflected schemes
k <- .5
g <- log(390)
zrs <- -(0:10)
ZRxgrsr.arl <- Vectorize(xgrsr.arl, "zr")
arls <- ZRxgrsr.arl(k, g, 0, zr=zrs)
data.frame(zrs, arls)

## Table 2 from Knoth (2006)
## original values are
# mu arl
# 0 697
# 0.5 33
# 1 10.4
# 1.5 6.2
# 2 4.4
# 2.5 3.5
# 3 2.9
#
k <- .5
g <- log(390)
zr <- -5 # see first example
mus <- (0:6)/2
Mxgrsr.arl <- Vectorize(xgrsr.arl, "mu")
arls <- round(Mxgrsr.arl(k, g, mus, zr=zr), digits=1)
data.frame(mus, arls)

XGRSR.arl <- Vectorize("xgrsr.arl", "g")
zr <- -6

## Table 2 from Moustakides et al. (2009)
## original values are
# gamma  A    ARL/E_infty(L)  SADD/E_1(L)
# 50     47.17  50.29        41.40
# 100    94.34 100.28        72.32
# 500    471.70 500.28       209.44
# 1000   943.41 1000.28      298.50
# 5000   4717.04 5000.24     557.87
#10000  9434.08 10000.17     684.17

theta <- .1
As2 <- c(47.17, 94.34, 471.7, 943.41, 4717.04, 9434.08)
gs2 <- log(As2)
arls0 <- round(XGRSR.arl(theta/2, gs2, 0, zr=-5, r=300, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs2, theta, zr=-5, r=300, MPT=TRUE), digits=2)
data.frame(As2, arls0, arls1)

## Table 3 from Moustakides et al. (2009)
## original values are
# gamma  A    ARL/E_infty(L)  SADD/E_1(L)
# 50     37.38  49.45        12.30
# 100    74.76  99.45        16.60
# 500    373.81 499.45       28.05
# 1000   747.62 999.45       33.33
# 5000   3738.08 4999.45     45.96
#10000  7476.15 9999.24      51.49

theta <- .5
As3 <- c(37.38, 74.76, 373.81, 747.62, 3738.08, 7476.15)
gs3 <- log(As3)
arls0 <- round(XGRSR.arl(theta/2, gs3, 0, zr=-5, r=70, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs3, theta, zr=-5, r=70, MPT=TRUE), digits=2)
data.frame(As3, arls0, arls1)

## Table 4 from Moustakides et al. (2009)
## original values are
# gamma  A    ARL/E_infty(L)  SADD/E_1(L)
# 50     28.02  49.78        4.98
# 100    56.04  99.79        6.22
# 500    280.19 499.79       9.30
# 1000   560.37 999.79      10.66
# 5000   2801.85 5000.93    13.86
#10000  5603.70 9999.87     15.24
theta <- 1
As4 <- c(28.02, 56.04, 280.19, 560.37, 2801.85, 5603.7)
gs4 <- log(As4)

arls0 <- round(XGRSR.arl(theta/2, gs4, 0, zr=-6, r=40, MPT=TRUE), digits=2)
arls1 <- round(XGRSR.arl(theta/2, gs4, theta, zr=-6, r=40, MPT=TRUE), digits=2)
data.frame(As4, arls0, arls1)

---

**xgrsr.crit**

*Compute alarm thresholds for Shiryaev-Roberts schemes*

**Description**

Computation of the alarm thresholds (alarm limits) for Shiryaev-Roberts schemes monitoring normal mean.

**Usage**

xgrsr.crit(k, L0, mu0 = 0, zr = 0, hs = NULL, sided = "one", MPT = FALSE, r = 30)

**Arguments**

- **k**: reference value of the Shiryaev-Roberts scheme.
- **L0**: in-control ARL.
- **mu0**: in-control mean.
- **zr**: reflection border to enable the numerical algorithms used here.
- **hs**: so-called headstart (enables fast initial response). If hs=NULL, then the classical headstart -Inf is used (corresponds to 0 for the non-log scheme).
- **sided**: distinguishes between one- and two-sided schemes by choosing "one" and "two", respectively. Currently only one-sided schemes are implemented.
- **MPT**: switch between the old implementation (FALSE) and the new one (TRUE) that considers the completed likelihood ratio. MPT contains the initials of G. Moustakides, A. Polunchenko and A. Tartakovsky.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1.

**Details**

xgrsr.crit determines the alarm threshold (alarm limit) for given in-control ARL L0 by applying secant rule and using xgrsr.arl().

**Value**

Returns a single value which resembles the alarm limit g.

**Author(s)**

Sven Knoth
References


See Also

xgrsr.arl for zero-state ARL computation.

Examples

```r
## Table 4 from Moustakides et al. (2009)
## original values are
# gamma/L0 A/exp(g)
# 50 28.02
# 100 56.04
# 500 280.19
# 1000 560.37
# 5000 2801.75
# 10000 5603.7
theta <- 1
zr <- -6
r <- 100
Lxgrsr.crit <- Vectorize("xgrsr.crit", "L0")
L0s <- c(50, 100, 500, 1000, 5000, 10000)
gs <- Lxgrsr.crit(theta/2, L0s, zr=zr, r=r)
data.frame(L0s, gs, A=round(exp(gs), digits=2))
```

---

**xsewma.arl**

Compute ARLs of simultaneous EWMA control charts (mean and variance charts)

**Description**

Computation of the (zero-state) Average Run Length (ARL) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance $S^2$) monitoring normal mean and variance.

**Usage**

```r
xsewma.arl(lx, cx, ls, csu, df, mu, sigma, hsx=0, Nx=40, csl=0, hss=1, Ns=40, s2.on=TRUE, sided="upper", qm=30)
```

**Arguments**

- **lx**: smoothing parameter lambda of the two-sided mean EWMA chart.
- **cx**: control limit of the two-sided mean EWMA control chart.
- **ls**: smoothing parameter lambda of the variance EWMA chart.
csu upper control limit of the variance EWMA control chart.
df actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
mu true mean.
sigma true standard deviation.
hsx so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
x dimension of the approximating matrix of the mean chart.
csl lower control limit of the variance EWMA control chart; default value is 0; not considered if sided is "upper".
hss headstart (enables fast initial response) of the variance chart.
x dimension of the approximating matrix of the variance chart.
s2.on distinguishes between $S^2$ and $S$ chart.
sided distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at $c_1$ – the actual value of $c_1$ is not used), "Rupper" (upper chart with reflection at $c_1$), "Rlower" (lower chart with reflection at $c_1$), and "two" (two-sided chart), respectively.
qm number of quadrature nodes used for the collocation integrals.

Details

xsewma.arl determines the Average Run Length (ARL) by an extension of Gan’s (derived from ideas already published by Waldmann) algorithm. The variance EWMA part is treated similarly to the ARL calculation method deployed for the single variance EWMA charts in Knoth (2005), that is, by means of collocation (Chebyshev polynomials). For more details see Knoth (2007).

Value

Returns a single value which resembles the ARL.

Author(s)

Sven Knoth

References


**See Also**

`xewma.arl` and `sewma.arl` for zero-state ARL computation of single mean and variance EWMA control charts, respectively.

**Examples**

```r
## Knoth (2007)
## collocation results in Table 1
## Monte Carlo with 10^9 replicates: 252.307 +/- 0.0078

# process parameters
mu <- 0
sigma <- 1
# subgroup size n=5, df=n-1
df <- 4
# lambda of mean chart
lx <- .134
# c_mu* = .345476571 = cx/sqrt(n) * sqrt(lx/(2-lx))
cx <- .345476571*sqrt(df+1)/sqrt(lx/(2-lx))
# lambda of variance chart
ls <- .1
# c_sigma = .477977
csu <- 1 + .477977
# matrix dimensions for mean and variance part
Nx <- 25
Ns <- 25
# mode of variance chart
SIDED <- "upper"

arl <- xsewma.arl(lx, cx, ls, csu, df, mu, sigma, Nx=Nx, Ns=Ns, sided=SIDED)
arl
```

---

**xsewma.crit**

`Compute critical values of simultaneous EWMA control charts (mean and variance charts)`

**Description**

Computation of the critical values (similar to alarm limits) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance $S^2$) monitoring normal mean and variance.

**Usage**

```r
xsewma.crit(lx, ls, L0, df, mu0=0, sigma0=1, cu=NULL, hsx=0, 
hs=1, s2.on=TRUE, sided="upper", mode="fixed", Nx=30, Ns=40, qm=30)
```
Arguments

1x  smoothing parameter lambda of the two-sided mean EWMA chart.
1s  smoothing parameter lambda of the variance EWMA chart.
L0  in-control ARL.
mu0 in-control mean.
sigma0  in-control standard deviation.
cu  for two-sided (sided="two") and fixed upper control limit (mode="fixed") a value larger than sigma0 has to been given, for all other cases cu is ignored.
hsx  so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
hss  headstart (enables fast initial response) of the variance chart.
df  actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
s2.on distinguishes between $S^2$ and $S$ chart.
sided distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at cl – the actual value of cl is not used), "Rupper" (upper chart with reflection at cl), "Rlower" (lower chart with reflection at cu), and "two" (two-sided chart), respectively.
mode  only deployed for sided="two" – with "fixed" an upper control limit (see cu) is set and only the lower is determined to obtain the in-control ARL L0, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
Nx  dimension of the approximating matrix of the mean chart.
Ns  dimension of the approximating matrix of the variance chart.
qm  number of quadrature nodes used for the collocation integrals.

Details

xsewma.crit determines the critical values (similar to alarm limits) for given in-control ARL L0 by applying secant rule and using xsewma.arl(). In case of sided="two" and mode="unbiased" a two-dimensional secant rule is applied that also ensures that the maximum of the ARL function for given standard deviation is attained at sigma0. See Knoth (2007) for details and application.

Value

Returns the critical value of the two-sided mean EWMA chart and the lower and upper controls limit cl and cu of the variance EWMA chart.

Author(s)

Sven Knoth
References


See Also

`xsewma.arl` for calculation of ARL of simultaneous EWMA charts.

Examples

```r
## Knoth (2007)
## results in Table 2

# subgroup size n=5, df=n-1
df <- 4
# lambda of mean chart
lx <- .134
# lambda of variance chart
ls <- .1
# in-control ARL
L0 <- 252.3
# matrix dimensions for mean and variance part
Nx <- 25
Ns <- 25
# mode of variance chart
SIDED <- "upper"

crit <- xsewma.crit(lx, ls, L0, df, sided=SIDED, Nx=Nx, Ns=Ns)
crit

## output as used in Knoth (2007)
crit["cx"]/sqrt(df+1)*sqrt(lx/(2-lx))
crit["cu"] - 1
```

---

**xsewma.q**

*Compute critical values of simultaneous EWMA control charts (mean and variance charts) for given RL quantile*

**Description**

Computation of the critical values (similar to alarm limits) for different types of simultaneous EWMA control charts (based on the sample mean and the sample variance $S^2$) monitoring normal mean and variance.

**Usage**

`xsewma.q(lx, cx, ls, csu, df, alpha, mu, sigma, hsx=0, Nx=40, csl=0, hss=1, Ns=40, sided="upper", qm=30)`
\texttt{xsewma.q.crit(lx, ls, L0, alpha, df, mu0=0, sigma0=1, cu=NULL,}
\texttt{hsx=0, hss=1, sided="upper", mode="fixed", Nx=20, Ns=40, qm=30,}
\texttt{c.error=1e-12, a.error=1e-9)}

**Arguments**

- **\texttt{lx}** smoothing parameter lambda of the two-sided mean EWMA chart.
- **\texttt{cx}** control limit of the two-sided mean EWMA control chart.
- **\texttt{ls}** smoothing parameter lambda of the variance EWMA chart.
- **\texttt{csu}** for two-sided (\texttt{sided="two"}) and fixed upper control limit (\texttt{mode="fixed"},
  only for \texttt{xsewma.q.crit}) a value larger than \texttt{sigma0} has to been given, for all other cases \texttt{cu} is ignored. It is the upper control limit of the variance EWMA control chart.
- **\texttt{L0}** in-control RL quantile at level \texttt{alpha}.
- **\texttt{df}** actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- **\texttt{alpha}** quantile level.
- **\texttt{mu}** true mean.
- **\texttt{sigma}** true standard deviation.
- **\texttt{mu0}** in-control mean.
- **\texttt{sigma0}** in-control standard deviation.
- **\texttt{hsx}** so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
- **\texttt{Nx}** dimension of the approximating matrix of the mean chart.
- **\texttt{csl}** lower control limit of the variance EWMA control chart; default value is 0; not considered if \texttt{sided} is "upper".
- **\texttt{hss}** headstart (enables fast initial response) of the variance chart.
- **\texttt{Ns}** dimension of the approximating matrix of the variance chart.
- **\texttt{sided}** distinguishes between one- and two-sided two-sided EWMA-$S^2$ control charts by choosing "upper" (upper chart without reflection at cl – the actual value of of cl is not used).
- **\texttt{mode}** only deployed for \texttt{sided="two"} – with "fixed" an upper control limit (see \texttt{cu}) is set and only the lower is determined to obtain the in-control ARL \texttt{L0}, while with "unbiased" a certain unbiasedness of the ARL function is guaranteed (here, both the lower and the upper control limit are calculated).
- **\texttt{qm}** number of quadrature nodes used for the collocation integrals.
- **\texttt{c.error}** error bound for two succeeding values of the critical value during applying the secant rule.
- **\texttt{a.error}** error bound for the quantile level \texttt{alpha} during applying the secant rule.
Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann’s survival function iteration procedure and on Knoth (2007). \texttt{xsewma.q.crit} determines the critical values (similar to alarm limits) for given in-control RL quantile \(L_0\) at level \(\alpha\) by applying secant rule and using \texttt{xsewma.sf()}. In case of \texttt{sided=“two”} and \texttt{mode=“unbiased”} a two-dimensional secant rule is applied that also ensures that the maximum of the RL cdf for given standard deviation is attained at \(\sigma_0\).

Value

Returns a single value which resembles the RL quantile of order \(\alpha\) and the critical value of the two-sided mean EWMA chart and the lower and upper controls limit \(csl\) and \(csu\) of the variance EWMA chart, respectively.

Author(s)

Sven Knoth

References


See Also

\texttt{xsewma.arl} for calculation of ARL of simultaneous EWMA charts and \texttt{xsewma.sf} for the RL survival function.

Examples

```r
## will follow
```

---

\texttt{xsewma.sf} \hspace{1cm} \textit{Compute the survival function of simultaneous EWMA control charts (mean and variance charts)}

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring simultaneously normal mean and variance.

Usage

\texttt{xsewma.sf(n, lx, cx, ls, csu, df, mu, sigma, hsx=0, Nx=40, csl=0, hss=1, Ns=40, sided=“upper”, qm=30)}
Arguments

- **n**: calculate sf up to value n.
- **\( \lambda_x \)**: smoothing parameter lambda of the two-sided mean EWMA chart.
- **\( \lambda_s \)**: control limit of the two-sided mean EWMA control chart.
- **\( \lambda_s \)**: smoothing parameter lambda of the variance EWMA chart.
- **\( c_{su} \)**: upper control limit of the variance EWMA control chart.
- **\( df \)**: actual degrees of freedom, corresponds to subgroup size (for known mean it is equal to the subgroup size, for unknown mean it is equal to subgroup size minus one.
- **\( \mu \)**: true mean.
- **\( \sigma \)**: true standard deviation.
- **\( h_{sx} \)**: so-called headstart (enables fast initial response) of the mean chart – do not confuse with the true FIR feature considered in xewma.arl; will be updated.
- **\( N_x \)**: dimension of the approximating matrix of the mean chart.
- **\( c_{sl} \)**: lower control limit of the variance EWMA control chart; default value is 0; not considered if sided is "upper".
- **\( h_{ss} \)**: headstart (enables fast initial response) of the variance chart.
- **\( N_s \)**: dimension of the approximating matrix of the variance chart.
- **sided**: distinguishes between one- and two-sided two-sided EWMA-\( S^2 \) control charts by choosing "upper" (upper chart without reflection at \( c_l \) – the actual value of \( c_l \) is not used), "\( R_{upper} \)" (upper chart with reflection at \( c_l \)), "\( R_{lower} \)" (lower chart with reflection at \( c_u \)), and "two" (two-sided chart), respectively.
- **\( q_m \)**: number of quadrature nodes used for the collocation integrals.

Details

The survival function \( P(L>n) \) and derived from it also the cdf \( P(L\leqslant n) \) and the pmf \( P(L=n) \) illustrate the distribution of the EWMA run length. For large n the geometric tail could be exploited. That is, with reasonable large n the complete distribution is characterized. The algorithm is based on Waldmann’s survival function iteration procedure and on results in Knoth (2007).

Value

Returns a vector which resembles the survival function up to a certain point.

Author(s)

Sven Knoth

References


xshewhart.ar1.arl

See Also

xsewma.arl for zero-state ARL computation of simultaneous EWMA control charts.

Examples

## will follow

---

**xshewhart.ar1.arl**  
*Compute ARLs of modified Shewhart control charts for AR(1) data*

### Description

Computation of the (zero-state) Average Run Length (ARL) for modified Shewhart charts deployed to the original AR(1) data.

### Usage

```
xshewhart.ar1.arl(alpha, cS, delta=0, N1=50, N2=30)
```

### Arguments

- `alpha`: lag 1 correlation of the data.
- `cS`: critical value (alias to alarm limit) of the Shewhart control chart.
- `delta`: potential shift in the data (in-control mean is zero).
- `N1`: number of quadrature nodes for solving the ARL integral equation, dimension of the resulting linear equation system is `N1`.
- `N2`: second number of quadrature nodes for combining the probability density function of the first observation following the margin distribution and the solution of the ARL integral equation.

### Details

Following the idea of Schmid (1995), 1-`alpha` times the data turns out to be an EWMA smoothing of the underlying AR(1) residuals. Hence, by combining the solution of the EWMA ARL integral equation and the stationary distribution of the AR(1) data (normal distribution is assumed) one gets easily the overall ARL.

### Value

It returns a single value resembling the zero-state ARL of a modified Shewhart chart.

### Author(s)

Sven Knoth
**References**


**See Also**

`xewma.arl` for zero-state ARL computation of EWMA control charts.

**Examples**

```r
## Table 1 in Kramer/Schmid (2000)

cS <- 3.09023
a <- seq(0, 4, by=.5)
row1 <- row2 <- row3 <- NULL
for ( i in 1:length(a) ) {
  row1 <- c(row1, round(xshewhart.ar1.arl( 0.4, cS, delta=a[i]), digits=2))
  row2 <- c(row2, round(xshewhart.ar1.arl( 0.2, cS, delta=a[i]), digits=2))
  row3 <- c(row3, round(xshewhart.ar1.arl(-0.2, cS, delta=a[i]), digits=2))
}
results <- rbind(row1, row2, row3)
results
# original values are
# row1 515.44 215.48 61.85 21.63 9.19 4.58 2.61 1.71 1.29
# row2 502.56 204.97 56.72 19.13 7.95 3.97 2.33 1.59 1.25
# row3 502.56 201.41 54.05 17.42 6.89 3.37 2.03 1.46 1.20
```

---

**xshewhartrunsrules.arl**

*Compute ARLs of Shewhart control charts with and without runs rules*

**Description**

Computation of the (zero-state and steady-state) Average Run Length (ARL) for Shewhart control charts with and without runs rules monitoring normal mean.

**Usage**

```r
xshewhartrunsrules.arl(mu, c = 1, type = "12")

xshewhartrunsrules.crit(L0, mu = 0, type = "12")
```
Arguments

\( \mu \)  
true mean.

\( \mu_0 \)  
pre-defined in-control ARL, that is, determine \( c \) so that the mean number of observations until a false alarm is \( \mu_0 \).

\( \mu_1, \mu_0 \)  
for the steady-state ARL two means are specified, \( \mu_0 \) is the in-control one and usually equal to 0, and \( \mu_1 \) must be given.

\( c \)  
normalizing constant to ensure specific alarming behavior.

\( \text{type} \)  
controls the type of Shewhart chart used, seed details section.

Details

\texttt{xshewhartrunsrules.arl} determines the zero-state Average Run Length (ARL) based on the Markov chain approach given in Champ and Woodall (1987). \texttt{xshewhartrunsrules.matrix} provides the corresponding transition matrix that is also used in \texttt{xshewhartrunsrules.arl} (ARL for control charting drift). \texttt{xshewhartrunsrules.crit} allows to find the normalization constant \( c \) to attain a fixed in-control ARL. Typically this is needed to calibrate the chart. With \texttt{xshewhartrunsrules.ad} the steady-state ARL is calculated. With the argument \text{type} certain runs rules could be set. The following list gives an overview.

- "1" The classical Shewhart chart with \( \pm 3 c \) sigma control limits (\( c \) is typically equal to 1 and can be changed by the argument \( c \)).
- "12" The classic and the following runs rule: 2 of 3 are beyond \( \pm 2 c \) sigma on the same side of the chart.
- "13" The classic and the following runs rule: 4 of 5 are beyond \( \pm 1 c \) sigma on the same side of the chart.
- "14" The classic and the following runs rule: 8 of 8 are on the same side of the chart (referring to the center line).

Value

Returns a single value which resembles the zero-state or steady-state ARL. \texttt{xshewhartrunsrules.matrix} returns a matrix.

Author(s)

Sven Knoth

References

See Also

xShewhartRunsRules.arl for zero-state ARL of Shewhart control charts with or without runs rules under drift.

Examples

```r
## Champ/Woodall (1987)
## Table 1
mus <- (0:15)/5
MxShewhartRunsRules.arl <- Vectorize(xShewhartRunsRules.arl, "mu")
# standard (1 of 1 beyond 3 sigma) Shewhart chart without runs rules
C1 <- round(MxShewhartRunsRules.arl(mus, type="1"), digits=2)
# standard + runs rule: 2 of 3 beyond 2 sigma on the same side
C12 <- round(MxShewhartRunsRules.arl(mus, type="12"), digits=2)
# standard + runs rule: 4 of 5 beyond 1 sigma on the same side
C13 <- round(MxShewhartRunsRules.arl(mus, type="13"), digits=2)
# standard + runs rule: 8 of 8 on the same side of the center line
C14 <- round(MxShewhartRunsRules.arl(mus, type="14"), digits=2)

## original results are
# mus C1 C12 C13 C14
# 0.0 370.40 225.44 166.05 152.73
# 0.2 308.43 177.56 120.70 110.52
# 0.4 200.08 104.46 63.88 59.76
# 0.6 119.67 57.92 33.99 33.64
# 0.8 71.55 33.12 19.78 21.07
# 1.0 43.89 20.01 12.66 14.58
# 1.2 27.82 12.81 8.84 10.90
# 1.4 18.25 8.69 6.62 8.60
# 1.6 12.38 6.21 5.24 7.03
# 1.8 8.69 4.66 4.33 5.85
# 2.0 6.30 3.65 3.68 4.89
# 2.2 4.72 2.96 3.18 4.08
# 2.4 3.65 2.48 2.78 3.38
# 2.6 2.90 2.13 2.43 2.81
# 2.8 2.38 1.87 2.14 2.35
# 3.0 2.00 1.68 1.89 1.99
data.frame(mus, C1, C12, C13, C14)

## plus calibration, i. e. L0=250 (the maximal value for "14" is 255!)
L0 <- 250
c1 <- xShewhartRunsRules.crit(L0, type = "1")
c12 <- xShewhartRunsRules.crit(L0, type = "12")
c13 <- xShewhartRunsRules.crit(L0, type = "13")
c14 <- xShewhartRunsRules.crit(L0, type = "14")
C1 <- round(MxShewhartRunsRules.arl(mus, c=c1, type="1"), digits=2)
C12 <- round(MxShewhartRunsRules.arl(mus, c=c12, type="12"), digits=2)
C13 <- round(MxShewhartRunsRules.arl(mus, c=c13, type="13"), digits=2)
C14 <- round(MxShewhartRunsRules.arl(mus, c=c14, type="14"), digits=2)
data.frame(mus, C1, C12, C13, C14)
```
## and the steady-state ARL

\[
M_{\text{xshewhartrunsrules.ad}} \leftarrow \text{Vectorize}(\text{xshewhartrunsrules.ad}, \text{"mu1"})
\]

\[
\begin{align*}
C_1 & \leftarrow \text{round}(M_{\text{xshewhartrunsrules.ad}}(\text{mus}, c=c_{1}, \text{type}=\text{"1"}), \text{digits}=2) \\
C_{12} & \leftarrow \text{round}(M_{\text{xshewhartrunsrules.ad}}(\text{mus}, c=c_{12}, \text{type}=\text{"12"}), \text{digits}=2) \\
C_{13} & \leftarrow \text{round}(M_{\text{xshewhartrunsrules.ad}}(\text{mus}, c=c_{13}, \text{type}=\text{"13"}), \text{digits}=2) \\
C_{14} & \leftarrow \text{round}(M_{\text{xshewhartrunsrules.ad}}(\text{mus}, c=c_{14}, \text{type}=\text{"14"}), \text{digits}=2)
\end{align*}
\]

\[
\text{data.frame(mus, C_1, C_{12}, C_{13}, C_{14})}
\]

---

**xtcusum.arl**

*Compute ARLs of CUSUM control charts*

**Description**

Computation of the (zero-state) Average Run Length (ARL) for different types of CUSUM control charts monitoring normal mean.

**Usage**

\[
\text{xtcusum.arl}(k, h, df, mu, hs = 0, \text{sided}=\text{"one"}, \text{mode}=\text{"tan"}, r=30)
\]

**Arguments**

- **k**: reference value of the CUSUM control chart.
- **h**: decision interval (alarm limit, threshold) of the CUSUM control chart.
- **df**: degrees of freedom – parameter of the t distribution.
- **mu**: true mean.
- **hs**: so-called headstart (give fast initial response).
- **sided**: distinguish between one- and two-sided CUSUM schemes by choosing "one" and "two", respectively.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \(r+1\).
- **mode**: Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.

**Details**

xtcusum.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature.

**Value**

Returns a single value which resembles the ARL.

**Author(s)**

Sven Knoth
xtewma.ad

References
D. Brook, D. A. Evans (1972), An approach to the probability distribution of cusum run length, Biometrika 59, 539-548.

See Also
xtewma.arl for zero-state ARL computation of EWMA control charts and xtcusum.arl for the zero-state ARL of CUSUM for normal data.

Examples
## will follow

xtewma.ad(l, c, df, mu1, mu0=0, zr=0, z0=0, sided="one", limits="fix", steady.state.mode="conditional", mode="tan", r=40)

Arguments
1 smoothing parameter lambda of the EWMA control chart.
c critical value (similar to alarm limit) of the EWMA control chart.
df degrees of freedom – parameter of the t distribution.
mu1 in-control mean.
mu0 out-of-control mean.
zr reflection border for the one-sided chart.
z0

restarting value of the EWMA sequence in case of a false alarm in steady.state.mode="cyclical".

sided
distinguishes between one- and two-sided two-sided EWMA control chart by choosing "one" and "two", respectively.

limits
distinguishes between different control limits behavior.

steady.state.mode
distinguishes between two steady-state modes – conditional and cyclical.

mode
Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.

r
number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).

Details
xtewma.ad determines the steady-state Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature and using the power method for deriving the largest in magnitude eigenvalue and the related left eigenfunction.

Value
Returns a single value which resembles the steady-state ARL.

Author(s)
Sven Knoth

References

See Also
xtewma.arl for zero-state ARL computation and xewma.ad for the steady-state ARL for normal data.

Examples
## will follow
xtewma.arl

Compute ARLs of EWMA control charts, t distributed data

Description

Computation of the (zero-state) Average Run Length (ARL) for different types of EWMA control charts monitoring the mean of t distributed data.

Usage

xtewma.arl(l,c,df,mu,zr=0,hs=0,sided="two",limits="fix",mode="tan",q=1,r=40)

Arguments

- **l**: smoothing parameter lambda of the EWMA control chart.
- **c**: critical value (similar to alarm limit) of the EWMA control chart.
- **df**: degrees of freedom – parameter of the t distribution.
- **mu**: true mean.
- **zr**: reflection border for the one-sided chart.
- **hs**: so-called headstart (enables fast initial response).
- **sided**: distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
- **limits**: distinguishes between different control limits behavior.
- **mode**: Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.
- **q**: change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For \( q > 1 \) and \( \mu! = 0 \) conditional delays, that is, \( E_q(L - q + 1 | L \geq q) \), will be determined. Note that \( \mu_0=0 \) is implicitly fixed.
- **r**: number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-sided) or \( r \) (two-sided).

Details

In case of the EWMA chart with fixed control limits, xtewma.arl determines the Average Run Length (ARL) by numerically solving the related ARL integral equation by means of the Nystroem method based on Gauss-Legendre quadrature. If limits is "vacl", then the method presented in Knoth (2003) is utilized. Other values (normal case) for limits are not yet supported.

Value

Except for the fixed limits EWMA charts it returns a single value which resembles the ARL. For fixed limits charts, it returns a vector of length \( q \) which resembles the ARL and the sequence of conditional expected delays for \( q=1 \) and \( q>1 \), respectively.
Author(s)

Sven Knoth

References


See Also

*xewma.arl* for zero-state ARL computation of EWMA control charts in the normal case.

Examples

```r
## Borror/Montgomery/Runger (1999), Table 3
lambda <- 0.1
cE <- 2.703
df <- c(4, 6, 8, 10, 15, 20, 30, 40, 50)
L0 <- rep(NA, length(df))
for (i in 1:length(df)) {
  L0[i] <- round(xtewma.arl(lambda, cE*sqrt(df[i]/(df[i]-2)), df[i], 0), digits=0)
}
data.frame(df, L0)
```

xtewma.q  
*Compute RL quantiles of EWMA control charts*

Description

Computation of quantiles of the Run Length (RL) for EWMA control charts monitoring normal mean.
Usage

xtewma.q(l, c, df, mu, alpha, zr=0, hs=0, sided="two", limits="fix", mode="tan", q=1, r=40)

xtewma.q.crit(l, L0, df, mu, alpha, zr=0, hs=0, sided="two", limits="fix", mode="tan", r=40, c.error=1e-12, a.error=1e-9, OUTPUT=FALSE)

Arguments

1  smoothing parameter lambda of the EWMA control chart.
c  critical value (similar to alarm limit) of the EWMA control chart.
df degrees of freedom – parameter of the t distribution.
mu true mean.
alpha quantile level.
zr reflection border for the one-sided chart.
hs so-called headstart (enables fast initial response).
sided distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.
limits distinguishes between different control limits behavior.
mode Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.
q change point position. For q = 1 and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state ARLs for the in-control and out-of-control case, respectively, are calculated. For q > 1 and \( \mu \neq 0 \) conditional delays, that is, \( E_q(L - q + 1 | L \geq) \), will be determined. Note that mu0=0 is implicitly fixed.
r number of quadrature nodes, dimension of the resulting linear equation system is equal to r+1 (one-sided) or r (two-sided).
L0 in-control quantile value.
c.error error bound for two succeeding values of the critical value during applying the secant rule.
a.error error bound for the quantile level alpha during applying the secant rule.
OUTPUT activate or deactivate additional output.

Details

Instead of the popular ARL (Average Run Length) quantiles of the EWMA stopping time (Run Length) are determined. The algorithm is based on Waldmann's survival function iteration procedure. If limits is "vacl", then the method presented in Knoth (2003) is utilized. For details see Knoth (2004).

Value

Returns a single value which resembles the RL quantile of order q.
xtewma.sf

Author(s)

Sven Knoth

References


See Also

xtewma.q for RL quantile computation of EWMA control charts in the normal case.

Examples

```#
## will follow
```

xtewma.sf  Compute the survival function of EWMA run length

Description

Computation of the survival function of the Run Length (RL) for EWMA control charts monitoring normal mean.

Usage

```r
xtewma.sf(l, c, df, mu, n, zr=0, hs=0, sided="two", limits="fix", mode="tan", q=1, r=40)
```

Arguments

- `l`: smoothing parameter lambda of the EWMA control chart.
- `c`: critical value (similar to alarm limit) of the EWMA control chart.
- `df`: degrees of freedom – parameter of the t distribution.
- `mu`: true mean.
- `n`: calculate sf up to value n.
- `zr`: reflection border for the one-sided chart.
- `hs`: so-called headstart (enables fast initial response).
sided
distinguishes between one- and two-sided EWMA control chart by choosing "one" and "two", respectively.

limits
distinguishes between different control limits behavior.

mode
Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.

q
change point position. For \( q = 1 \) and \( \mu = \mu_0 \) and \( \mu = \mu_1 \), the usual zero-state situation for the in-control and out-of-control case, respectively, are calculated. Note that \( \mu_0=0 \) is implicitly fixed.

r
number of quadrature nodes, dimension of the resulting linear equation system is equal to \( r+1 \) (one-sided) or \( r \) (two-sided).

Details
The survival function \( P(L>n) \) and derived from it also the cdf \( P(L<=n) \) and the pmf \( P(L=n) \) illustrate the distribution of the EWMA run length. For large \( n \) the geometric tail could be exploited. That is, with reasonable large \( n \) the complete distribution is characterized. The algorithm is based on Waldmann’s survival function iteration procedure. For varying limits and for change points after 1 the algorithm from Knoth (2004) is applied. For details see Knoth (2004).

Value
Returns a vector which resembles the survival function up to a certain point.

Author(s)
Sven Knoth

References

See Also
*xtewma.sf* for survival function computation of EWMA control charts in the normal case.

Examples
```r
## will follow```
xtshewhart.ar1.arl

Compute ARLs of modified Shewhart control charts for AR(1) data with Student t residuals

Description

Computation of the (zero-state) Average Run Length (ARL) for modified Shewhart charts deployed to the original AR(1) data where the residuals follow a Student t distribution.

Usage

xtshewhart.ar1.arl(alpha, cS, df, delta=0, N1=50, N2=30, N3=2*N2, INFI=10, mode="tan")

Arguments

alpha  
lag 1 correlation of the data.

cS  
critical value (alias to alarm limit) of the Shewhart control chart.

df  
degrees of freedom – parameter of the t distribution.

delta  
potential shift in the data (in-control mean is zero).

N1  
number of quadrature nodes for solving the ARL integral equation, dimension of the resulting linear equation system is N1.

N2  
second number of quadrature nodes for combining the probability density function of the first observation following the margin distribution and the solution of the ARL integral equation.

N3  
third number of quadrature nodes for solving the left eigenfunction integral equation to determine the margin density (see Andel/Hrach, 2000), dimension of the resulting linear equation system is N3.

INFI  
substitute of Inf – the left eigenfunction integral equation is defined on the whole real axis; now it is reduced to (-INFI, INFI).

mode  
Controls the type of variables substitution that might improve the numerical performance. Currently, "identity", "sin", "sinh", and "tan" (default) are provided.

Details

Following the idea of Schmid (1995), 1-alpha times the data turns out to be an EWMA smoothing of the underlying AR(1) residuals. Hence, by combining the solution of the EWMA ARL integral equation and the stationary distribution of the AR(1) data (here Student t distribution is assumed) one gets easily the overall ARL.

Value

It returns a single value resembling the zero-state ARL of a modified Shewhart chart.
Author(s)
Sven Knoth

References


See Also
xtewma.arl for zero-state ARL computation of EWMA control charts in case of Student t distributed data.

Examples

```r
## will follow
```
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