Package ‘spray’
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Type Package

Title Sparse Arrays and Multivariate Polynomials

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Maintainer Robin K. S. Hankin <hankin.robin@gmail.com>

Description Sparse arrays interpreted as multivariate polynomials.

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Depends methods

Suggests polynom, testthat, covr

Imports Rcpp (&gt;= 0.12.3), partitions, magic, mathjaxr, disordR (&gt;= 0.9-6)

LinkingTo Rcpp

URL https://github.com/RobinHankin/spray

BugReports https://github.com/RobinHankin/spray/issues

RdMacros mathjaxr

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Description

Functionality for sparse arrays, with emphasis on their interpretation as multivariate polynomials.

Details

Base R has the capability of dealing with arbitrary dimensioned numerical arrays, with the array class.

A sparse array is a type of array in which nonzero elements are stored along with an index vector describing their coordinates. This allows for efficient storage and manipulation as base arrays often require the storing of many zero elements which consume computational and memory resources.

In the package, sparse arrays are represented as objects of class spray. They use the C++ standard template library (STL) map class, with keys being (unsigned) integer vectors, and values floats.

One natural application of sparse arrays, for which the package was written, is multivariate polynomials and the package vignette presents an extended discussion. Note that other interpretations exist: the stokes and weyl packages interpret spray objects as differential forms and elements of a Weyl algebra respectively.

Author(s)

Robin K. S. Hankin

Examples

# define a spray using a matrix of indices and a vector of values:
M <- matrix(sample(0:3,21,replace=TRUE),ncol=3)
a <- spray(M,sample(7))

# there are many pre-defined simple sprays:
b <- homog(3,4)

# arithmetic operators work:
a + 2*b
a - a*b^2/4
a=b
# we can sum over particular dimensions:
assum(a+b,1)

# differentiation is supported:
deriv(a^6,2)

# extraction and replacement work as expected:

b[1,2,1]
b[1,2,1,drop=TRUE]
b[diag(3)] <- 3

---

**arity**  

*The arity of a spray object*

---

**Description**

The arity of a spray object: the number of indices needed to retrieve an entry, or the number of columns in the index matrix.

**Usage**

```r
arity(S)
```

**Arguments**

- `S`: a spray object

**Value**

Returns an integer

**Author(s)**

Robin K. S. Hankin

**Examples**

```r
(a <- rspray())
arity(a)
```
as.array

Coerce spray objects to arrays

Description

Coerces spray objects to arrays. Includes off-by-one functionality via option offbyone.

Usage

### S3 method for class 'spray'

as.array(x, offbyone=FALSE, compact=FALSE, ...)

### S3 method for class 'spray'

dim(x)

Arguments

- **x** spray object
- **offbyone** Boolean with default FALSE meaning to interpret the index entries as positions in their dimension, and TRUE meaning to add one to index values so that zero entries appear in the first place
- **compact** Boolean with default FALSE meaning to translate the spray as is, and TRUE meaning to add constants to each column of the index matrix so that the resulting array is as small as possible
- **...** Further arguments, currently ignored

Details

Argument offbyone defaults to FALSE; but if it is set to TRUE, it effectively adds one from the index matrix, so a zero entry in the index matrix means the first position in that dimension.

After the subtraction, if performed, the function will not operate if any index is less than 1.

Value

Returns an array of dimension dim(S). The “meat” of the function is

```r
out <- array(0, dS)
out[ind] <- coeffs(S)
```

Author(s)

Robin K. S. Hankin

Examples

```r
(M <- matrix(sample(0:4,28,replace=TRUE),ncol=4))
(S <- spray(M,sample(7),addrepeats=TRUE))
as.array(S,offbyone=TRUE)  # a large object! sprays are terse

S <- spray(matrix(sample(1:4,28,replace=TRUE),ncol=4),sample(7))
A <- as.array(S)  # S has no zero indices [if it did, we would need to use offbyone=TRUE]
stopifnot(all(S[index(S),drop=TRUE] == A[index(S)]))
```
as.character

**Description**
Coerces spray objects to a character string or disord character vector.

**Usage**
```r
## S3 method for class 'spray'
as.character(x, ..., split=FALSE)
```

**Arguments**
- `x` spray object
- `...` Further arguments, currently ignored
- `split` Boolean with default FALSE meaning to return a length-one character vector, and TRUE meaning to return a disord object with elements being terms, coerced to character

**Details**
The method uses `print_spray_polyform()` and as such is sensitive to option `sprayvars`, but not `polyform`.

**Author(s)**
Robin K. S. Hankin

**Examples**
```r
as.character(rspray())
as.character(rspray(),split=TRUE)
```

---

as.function.spray

**Description**
Coerce a spray object to a function

**Usage**
```r
## S3 method for class 'spray'
as.function(x,...)
```

**Arguments**
- `x` spray object, interpreted as a multivariate polynomial
- `...` Further arguments, currently ignored
Value

Returns a function; this function returns a numeric vector.

Note

Coercion is possible even if some indices are zero or negative. The function is not vectorized in the arity of its argument.

Author(s)

Robin K. S. Hankin

Examples

\[
S \leftarrow \text{spray}(\text{matrix}(1:6,3,2),1:3)) \\
(f \leftarrow \text{as.function}(S)) \\
f(2:3) == 3 \times 2^3 \times 3^6 + 2 \times 2^2 \times 3^5 + 1 \times 2^1 \times 3^4 \quad \# \text{ should be TRUE}
\]

\[
S1 \leftarrow \text{spray}(\text{matrix}(\text{sample}(-2:2, \text{replace}=\text{TRUE},21), \text{ncol}=3), \text{rnorm}(7), \text{addrepeats}=\text{TRUE}) \\
S2 \leftarrow \text{spray}(\text{matrix}(\text{sample}(-2:2, \text{replace}=\text{TRUE},15), \text{ncol}=3), \text{rnorm}(5), \text{addrepeats}=\text{TRUE})
\]

\[
f1 \leftarrow \text{as.function}(S1) \\
f2 \leftarrow \text{as.function}(S2)
\]

\[
f3 \leftarrow \text{as.function}(S1 \times S2)
\]

\[
x \leftarrow 4:6
\]

\[
f1(x) \times f2(x) - f3(x) \quad \# \text{ should be zero}
\]

\[
\# \text{ coercion is vectorized:}
\]

\[
f1(\text{matrix}(1:33, \text{ncol}=3))
\]

---

**asum**  
*Sum over dimension margins*

Description

Sum over specified dimension margins.

Usage

```
## S3 method for class 'spray'
asum(S, dims, drop=TRUE, ...) 
asum_inverted(S, dims) 
process_dimensions(S,dims)
```
constant

Arguments

- **S**: spray object
- **dims**: Vector of strictly positive integers corresponding to dimensions to be summed over
- **drop**: Boolean, with default TRUE meaning to drop the summed dimensions, and FALSE meaning to retain them.
- **...**: Further arguments, currently ignored

Details

Function `asum.spray()` is the method for `asum()`. This takes a spray, and a vector of integers corresponding to dimensions to be summed over.

Function `asum_inverted()` is the same, but takes a vector of integers corresponding to dimensions not to sum over. This function is here because there is nice C++ idiom for it.

Function `process_dimensions()` ensures that the `dims` argument is consistent with the spray `S` and returns a cleaned version thereof.

Value

Returns a spray object.

Author(s)

Robin K. S. Hankin

Examples

```r
S <- spray(matrix(sample(0:2,60,replace=TRUE),ncol=3),addrepeats=TRUE)
S
asum(S,1)
asum(S,1:2)
asum(S,1:2,drop=FALSE)
asum(S,c(1,3)) == asum_inverted(S,2)
```

---

constant

Get or set the constant term of a spray object

Description

The constant term of a spray object is the coefficient corresponding to an index of all zeros. These functions get or set the constant of a spray object.
Usage

is.constant(x)
constant(x, drop=FALSE)
constant(x) <- value
drop(x)

Arguments

x Object of class spray
value Numeric value to set the constant coefficient to
drop Boolean, with default FALSE meaning to return a spray object and TRUE meaning to return a numeric value

Value

In function constant(), return the coefficient, or a constant multivariate polynomial, depending on the value of drop.

Note

The behaviour of the drop argument (sort of) matches that of the spray extractor method. Function drop() returns the elements of the coefficients.

Function constant() ensures that zero spray objects retain the argument’s arity.

It might have been better to call is.constant() is.scalar(), for consistency with the stokes and clifford packages. But this is not clear.

Author(s)

Robin K. S. Hankin

See Also

Extract

Examples

(S <- spray(partitions::blockparts(rep(2,3),3,TRUE)))

constant(S)
constant(S) <- 33
S
drop(constant(S, drop=FALSE))
Partial differentiation of spray objects

Description
Partial differentiation of spray objects interpreted as multivariate polynomials

Usage
```r
# S3 method for class 'spray'
deriv(expr, i, derivative = 1, ...)
deriv(S, orders)
```

Arguments
- `expr`: A spray object, interpreted as a multivariate polynomial
- `i`: Dimension to differentiate with respect to
- `derivative`: How many times to differentiate
- `...`: Further arguments, currently ignored
- `S`: spray object
- `orders`: The orders of the differentials

Details
Function `deriv.spray()` is the method for generic `spray()`; if `S` is a spray object, then `spray(S, i, n)` returns $\frac{\partial^n S}{\partial x_1^n} S^{(x_1, \ldots, x)}$.
Function `aderiv()` is the generalized derivative; if `S` is a spray of arity 3, then `aderiv(S, c(i,j,k))` returns $\frac{\partial^{i+j+k} S}{\partial x_1^i \partial x_2^j \partial x_3^k}$.

Value
Both functions return a spray object.

Author(s)
Robin K. S. Hankin

See Also
- `asum`

Examples
```r
(S <- spray(matrix(sample(-2:2,15,replace=TRUE),ncol=3),addrepeats=TRUE))
deriv(S,1)
deriv(S,2,2)
```

# differentiation is invariant under order:
aderiv(S,1:3) == deriv(deriv(deriv(S,1,2),2,3),3,3)

# Leibniz's rule:
S1 <- spray(matrix(sample(0:3,replace=TRUE,21),ncol=3),sample(7),addrepeats=TRUE)
S2 <- spray(matrix(sample(0:3,replace=TRUE,15),ncol=3),sample(5),addrepeats=TRUE)
S1*deriv(S2,1) + deriv(S1,1)*S2 == deriv(S1*S2,1)

# Generalized Leibniz:
aderiv(S1*S2,c(1,1,0)) == 
aderiv(S1,c(0,0,0))*aderiv(S2,c(1,1,0)) +
aderiv(S1,c(0,1,0))*aderiv(S2,c(1,0,0)) +
aderiv(S1,c(1,0,0))*aderiv(S2,c(0,1,0)) +
aderiv(S1,c(1,1,0))*aderiv(S2,c(0,0,0))

Extract.spray

Extract or Replace Parts of a spray

Description

Extract or replace subsets of sprays.

Usage

## S3 method for class 'spray'
S[...], drop=FALSE]

## S3 replacement method for class 'spray'
S[index, ...] <- value

Arguments

S        A spray object
index    elements to extract or replace
value    replacement value
...       Further arguments
drop    Boolean, with default FALSE meaning to return a spray object and TRUE meaning to drop the spray structure and return a numeric vector

Details

These methods should work as expected, although the off-by-one issue might be a gotcha. disordR discipline is enforced where appropriate.

In S[index, ...], argument drop is FALSE by default, in which case a spray object is returned. If drop is TRUE a numeric vector is returned, with elements corresponding to the rows of index. Compare coeffs(S), which returns a disord object; in S[index,drop=TRUE], the rows of index specify a unique order for the return value.

If a <- spray(diag(3)), for example, then idiom such as a[c(1,2,3)] cannot work, because one would like a[1,2,3] and a[1:3,2,3] to work.
If \( p \leq 1:3 \), then one might expect idiom such as \( S[1,,p,1:3] \) to work but this is problematic and a discussion is given in inst/missing_accessor.txt.

Functions \( \text{spray\_extract\_disord()} \) and \( \text{spray\_replace\_disord()} \) are low-level helper functions which implement idiom such as \( a[\text{coeffs}(a) < 3] \) and \( a[\text{coeffs}(a) < 3] \leftarrow 99 \).

Examples

\[
\begin{align*}
(a & \leftarrow \text{spray(diag(5)))} \\
& a[rbind(rep(1,5))] \leftarrow 5 \\
a & \\
& a[3,4,5,3,1] \ # \text{the NULL polynomial} \\
a[0,1,0,0,0] \\
a[0,1,0,0,0,\text{drop=TRUE}] \\
a[2,3:5,4,3,3] \leftarrow 9 \\
a & \\
& \text{options(polyform = TRUE)} \ # \text{print as a multivariate polynomial} \\
a & \\
& \text{options(polyform = FALSE)} \ # \text{print in sparse array form} \\
a & \\
&S2 & \leftarrow \text{spray(1-diag(5),11:15)} \\
&S3 & \leftarrow \text{spray(rbind(c(1,0,0,0,c(1,2,1,1,1)))))} \\
&S1 & \leftarrow 3 \\
&S1 & \leftarrow S2 \\
&S1[S3] \leftarrow 99 \\
&S1 & \\
&S & \leftarrow \text{rspray()} \\
&S[\text{coeffs}(S) > 4] \\
&S[\text{coeffs}(S) < 6] \leftarrow 99 \\
&S & 
\end{align*}
\]

**homog**

Various functions to create simple spray objects

**Description**

Various functions to create simple spray objects such as single-term, homogeneous, and constant multivariate polynomials.

**Usage**

\[
\begin{align*}
\text{product(power)} & \\
\text{homog(d, power=1)} & 
\end{align*}
\]
linear(x,power=1)
lone(n,d=n)
one(d)
asa.id(S)
xyz(d)

**Arguments**

- **d**: An integer; generally, the dimension or arity of the resulting spray object
- **power**: Integer vector of powers
- **x**: Numeric vector of coefficients
- **S**: A spray object
- **n**: In function lone(), the term to raise to power 1

**Value**

All functions documented here return a spray object

**Note**

The functions here are related to their equivalents in the `multipol` package, but are not exactly the same.

Function `zero()` is documented at `zero.Rd`, but is listed below for convenience.

**Author(s)**

Robin K. S. Hankin

**See Also**

`constant, zero`

**Examples**

```r
product(1:3)  #  x * y^2 * z^3
homog(3)  #  x + y + z
homog(3,2)  #  x^2 + xy + xz + y^2 + yz + z^2
linear(1:3)  #  1*x + 2*y + 3*z
linear(1:3,2)  #  1*x^2 + 2*y*z + 3*z^2
lone(3)  #  z
lone(2,3)  #  y
one(3)  #  1
zero(3)  #  0
xyz(3)  #  xyz
```
**knight**

Generating function for a chess knight and king

**Description**

Generating function for a chess knight and king on an arbitrarily-dimensioned chessboard

**Usage**

```r
knight(d=2)
king(d=2)
```

**Arguments**

- `d` Dimensionality of the board, defaulting to 2

**Value**

Returns the generating function of the piece in question.

**Note**

The pieces are forced to move; if they have the option of not moving, add 1 to the returned spray. The vignette contains a short discussion.

**Author(s)**

Robin K. S. Hankin

**Examples**

```r
knight()  # default 2D chess board
king()    # ditto

knight()^2  # generating function for two knight's moves

## How many ways can a knight return to its starting square in 6 moves?
constant(knight()^6)

## How many in 6 or fewer?
constant((1+knight())^6)

## Where does a randomly-moving knight end up?
d <- xyz(2)
kt <- (1+knight())*d^2/9
persp(1:25,1:25,as.array(d*kt^6))

## what is the probability that a 4D king is a knight's move from
## (0,0,0,0) after 6 moves?
sum(coeffs(((king(4)/80)^4)[knight(4)]))
```
nterms \hspace{1cm} \textit{Number of nonzero terms in a spray object}

\textbf{Description}

Number of nonzero terms in a spray object

\textbf{Usage}

\begin{verbatim}
nterms(x)
## S3 method for class 'spray'
length(x)
\end{verbatim}

\textbf{Arguments}

\begin{itemize}
\item \texttt{x} \hspace{1cm} Object of class spray
\end{itemize}

\textbf{Details}

Number of nonzero terms in a spray object. Function \texttt{length()} is defined so that \texttt{seq_along()} works as expected.

\textbf{Author(s)}

Robin K. S. Hankin

\textbf{Examples}

\begin{verbatim}
(a <- rspray())
nterms(a)

seq_along(a)
\end{verbatim}

\textbf{ooom} \hspace{1cm} \textit{One-over-one-minus for spray objects}

\textbf{Description}

One-over-one-minus for spray objects; the nearest to ‘division’ that we can get.

\textbf{Usage}

\begin{verbatim}
ooom(S, n)
\end{verbatim}

\textbf{Arguments}

\begin{itemize}
\item \texttt{S} \hspace{1cm} object of class spray
\item \texttt{n} \hspace{1cm} Order of the approximation
\end{itemize}
Details

Returns the Taylor expansion to order \( n \) of \( 1/(1 - S) \), that is, \( 1 + S + S^2 + S^3 + \cdots + S^n \).

Value

Returns a spray object of the same arity as \( S \).

Note

Uses Horner’s method for efficiency

Author(s)

Robin K. S. Hankin

Examples

\[
(x \leftarrow \text{spray(matrix(1)))}
\]
\[
\text{ooom(x,5)} \quad # \quad 1 + x + x^2 + x^3 + x^4 + x^5
\]

\[
(a \leftarrow \text{homog}(4,2))
\]
\[
d \leftarrow (1-a)\times\text{ooom}(a,3)
\]
\[
\text{constant(d)} \quad # \text{should be 1}
\]
\[
\text{rowSums(index(d))} \quad # \text{a single 0 and lots of 8s}
\]
Details

The function \texttt{Ops.spray()} passes unary and binary arithmetic operators ("+", "-", "\times", "/", "+", and "\times\times") to the appropriate specialist function.

The most interesting operators are "\times" and "+" which execute multivariate polynomial multiplication and addition respectively.

Testing for equality uses \texttt{spray\_eq\_spray()}. Note that \texttt{spray\_eq\_spray(S1,S2)} is algebraically equivalent to \texttt{is.zero(S1-S2)}, but faster (\texttt{FALSE} is returned as soon as a mismatch is found).

Value

The functions all return spray objects except "==", which returns a logical.

Author(s)

Robin K. S. Hankin

See Also

\texttt{ooom}

Examples

\begin{verbatim}
M <- matrix(sample(0:3,21,replace=TRUE),ncol=3)
a <- spray(M,sample(7))
b <- homog(3,4)

# arithmetic operators mostly work as expected:
a + 2*b
a - a*b^2/4
a+b

S1 <- spray(partitions::compositions(4,3))
S2 <- spray(diag(3)) # S2 = x+y+z

stopifnot( (S1+S2)^3 == S1^3 + 3*S1^2*S2 + 3*S1*S2^2 + S2^3 )
\end{verbatim}

\begin{verbatim}
\texttt{pmax} \hspace{1cm} Parallel maxima and minima for sprays
\end{verbatim}

Description

Parallel (pairwise) maxima and minima for sprays.
Usage

maxpair_spray(S1,S2)
minpair_spray(S1,S2)

## S3 method for class 'spray'
pmax(x, ...)

Arguments

x, S1, S2   Spray objects
...         spray objects to be compared

Details

Function maxpair_spray() finds the pairwise maximum for two sprays. Specifically, if \( S3 \leftarrow \text{maxpair_spray}(S1, S2) \), then \( S3[v] = \max(S1[v], S2[v]) \) for every index vector \( v \).

Function pmax.spray() is the method for the generic pmax(), which takes any number of arguments. If \( S3 \leftarrow \text{maxpair_spray}(S1, S2, ...), S3[v] = \max(S1[v], S2[v], ...) \) for every index vector \( v \).

Function pmax.spray() operates right-associatively:

\[
pmax(S1, S2, S3, S4) = f(f(S1, f(S2, f(S3, S4))))
\]

where \( f() \) is short for maxpair_spray(). So if performance is important, put the smallest spray (in terms of number of nonzero entries) last.

In these functions, a scalar is interpreted as a sort of global maximum. Thus if \( S3 \leftarrow \text{pmax}(S, x) \) we have \( S3[v] = \max(S[v], x) \) for every index \( v \). Observe that this operation is not defined if \( x > 0 \), for then there would be an infinity of \( v \) for which \( S3[v] \neq 0 \), an impossibility (or at least counter to the principles of a sparse array). Note also that \( x \) cannot have length > 1 as the elements of a spray object are stored in an arbitrary order, following disordR discipline.

Functions minpair_spray() and pmin.spray() are analogous. Note that minpair_spray(S1, S2) is algebraically equivalent to \(-\text{pmax_spray}(-S1, -S2)\); see the examples.

The value of pmax(S) is problematic. Suppose \( \text{all(coeffs}(S)<0) \); the current implementation returns pmax(S)==S but there is a case for returning the null polynomial.

Value

Returns a spray object

Author(s)

Robin K. S. Hankin

Examples

\[
S1 \leftarrow \text{rspray}(100, \text{vals}=\text{sample}(100)-50)\\
S2 \leftarrow \text{rspray}(100, \text{vals}=\text{sample}(100)-50)\\
S3 \leftarrow \text{rspray}(100, \text{vals}=\text{sample}(100)-50)
\]

# following comparisons should all be TRUE:

\[
jj \leftarrow \text{pmax}(S1, S2, S3)
\]
\[ jj == \maxpair\_spray(S1,\maxpair\_spray(S2,S3)) \]
\[ jj == \maxpair\_spray(\maxpair\_spray(S1,S2),S3) \]
\[ \pmax(S1,S2,S3) == -\pmin(-S1,-S2,-S3) \]
\[ \pmin(S1,S2,S3) == -\pmax(-S1,-S2,-S3) \]
\[ \pmax(S1,-\infty) == S1 \]
\[ \pmin(S1, \infty) == S2 \]

\[ \pmax(S1,-3) \]

\[ \text{## Not run:} \]
\[ \pmax(S1,3) \# \not \text{ defined} \]

\[ \text{## End(Not run)} \]

---

**print.spray**

Print methods for spray objects

**Description**

Print methods for spray objects with options for printing in matrix form or multivariate polynomial form

**Usage**

```r
## S3 method for class 'spray'
print(x, ...) 
print_spray_matrixform(S) 
print_spray_polyform(S,give=FALSE)
```

**Arguments**

- `x`<br> spray object
- `S`<br> spray object
- `give`<br> Boolean, with default FALSE meaning to print the value of `S`, and TRUE meaning to return a string (without nice formatting); used in `as.character.spray()`
- `...`<br> Further arguments (currently ignored)

**Details**

The `print.spray()` dispatches to helper functions `print_spray_matrixform()` and `print_spray_polyform()` depending on the value of option `polyform`; see the examples section.

Option `sprayvars` is a character vector with entries corresponding to the variable names for printing. The `sprayvars` option has no algebraic significance: all it does is affect the print method.

Note that printing a spray object (in either matrix form or polynomial form) generally takes much longer than calculating it.
Value

Returns its argument invisibly.

Note

There are a couple of hard-wired symbols for multiplication and equality which are defined near the top of the helper functions.

Author(s)

Robin K. S. Hankin

Examples

(a <- spray(diag(3)))

options(polyform = FALSE)
a^3

options(polyform = TRUE)
a^3

options(sprayvars=letters)
a <- diag(26)
spray(a)

## Following example from mpoly:
a[1 + cbind(0:25, 1:26) %% 26] <- 2
spray(a)

rspray

Random spray objects

Description

Creates random spray objects as quick-and-dirty examples of multivariate polynomials

Usage

rspray(n=9, vals = seq_len(n), arity = 3, powers = 0:2)

Arguments

n Number of distinct rows (maximum); repeated rows are merged (argument addrepeats is TRUE)
vals Values to use for coefficients
arity Arity of the spray; the number of columns in the index matrix
powers Set from which to sample the entries of the index matrix
Value

Returns a spray object

Note

If the index matrix contains repeated rows, the returned spray object will contain fewer than \( n \) entries

Author(s)

Robin K. S. Hankin

See Also

spray

Examples

rspray()

rspray(4)*rspray(3,rnorm(3))

rspray(3,arity=7,powers=-2:2)^3

rspray(1000,vals=rnorm(1000))

---

**spray**

Sparse arrays: spray objects

Description

Create, coerce, and test for sparse array objects

Usage

```
spray(M, x, addrepeats=FALSE)
spraymaker(L, addrepeats=FALSE, arity=ncol(L[[1]]))
is.spray(S)
as.spray(arg1, arg2, addrepeats=FALSE, offbyone=FALSE)
index(S)
coeffs(S)
coeffs(S) <- value
is_valid_spray(L)
```

Arguments

- **M**: Integer matrix with rows corresponding to index positions
- **x**: Numeric value with elements corresponding to spray entries
- **S**: Object to be tested for being a spray
spray

L A list, nominally of two elements (index matrix and value) which is to be tested for acceptability to be coerced to class spray

arg1, arg2 Various arguments to be coerced to a spray

addrepeats Boolean, with default FALSE meaning to check for repeated index rows and, if any are found, return an error

value In the assignment operator coeffs<-(), a disord object (or a length-one numeric vector), so that coeffs(S) <- x works as expected

offbyone In function as.spray(), when converting from an array. Argument offbyone is Boolean with default FALSE meaning to insert array elements in positions corresponding to index elements, and TRUE meaning to add one

arity In function spraymaker(), integer specifying the arity (number of columns of the index matrix L[[1]]); ignored if L is non-empty. See details

Details

Spray objects are sparse arrays interpreted as multivariate polynomials. They can be added and subtracted; "+" is interpreted as polynomial multiplication.

To create a spray object the user should use spray(), if a matrix of indices and vector of values is available, or as.spray() which tries hard to do the Right Thing (tm).

Function spraymaker() is the formal creator function, and it is written to take the output of the C++ routines and return a spray object. The reason this needs an arity argument is that C++ sometimes returns NULL (in lieu of a zero-row matrix, which it cannot deal with). In this case, we need some way to tell R the arity of the corresponding spray object.

Functions index() and coeffs() are accessor methods.

There is an extensive vignette available; type vignette("spray") at the command line.

Note

Function coeffs() was formerly known as value().

Technically, index() breaks disordR discipline.

Author(s)

Robin K. S. Hankin

See Also

Ops, spray-package

Examples

S <- spray(diag(5)) # missing second argument interpreted as '1'.
as.array(S,offbyone=TRUE) # zero indices interpreted as ones.

M <- matrix(1:5,6,5) # note first row matches the sixth row

## Not run: spray(M,1:6) # will not work because addrepeats is not TRUE

spray(M,1:6,addrepeats=TRUE) # 7=1:6
$S \leftarrow spray(matrix(1:7,5,7))$
$a \leftarrow as.array(S)$  # will not work if any(M<1)
$S1 \leftarrow as.spray(a)$
stopifnot(S==S1)

$a \leftarrow rspray(20)$
coeffs(a)[coeffs(a) %% 2 == 1] <- 99  # every odd coefficient -> 99

---

### spray-class

**Class** “spray”

**Description**

The formal S4 class for sprays.

**Objects from the Class**

Objects can be created by calls of the form `new("spray", ...)` but this is not encouraged. Use functions `spray()` or `as.spray()` instead.

**Slots**

*index*: Index matrix

*value*: Numeric vector holding coefficients

**Author(s)**

Robin K. S. Hankin

**See Also**

`spray`

---

### spray_cpp

**Low-level functions that call C++ source code**

**Description**

Low-level functions that call C++ source code, as detailed in the automatically generated `RcppExports.R` file.
Usage

spray_maker(M, d)
spray_add(M1, d1, M2, d2)
spray_mult(M1, d1, M2, d2)
spray_overwrite(M1, d1, M2, d2)
spray_accessor(M, d, Mindex)
spray_setter(M1, d1, M2, d2)
spray_equality(M1, d1, M2, d2)
spray_asm_include(M, d, n)
spray_asm_exclude(M, d, n)
spray_deriv(M, d, n)
spray_pmax(M1, d1, M2, d2)
spray_pmin(M1, d1, M2, d2)
spray_power(M, d, pow)
spray_spray_accessor()
spray_spray_add()
spray_spray_asm_exclude()
spray_spray_asm_include()
spray_spray_deriv()
spray_spray_equality()
spray_spray_maker()
spray_spray_mult()
spray_spray_overwrite()
spray_spray_pmax()
spray_spray_pmin()
spray_spray_setter()
spray_spray_power()

Arguments

M, M1, M2, Mindex  Integer valued matrices with rows corresponding to array indices
d, d1, d2          Vector of values corresponding to nonzero array entries
n                   Integer vector corresponding to dimensions to sum over for the sum functions
pow                 Nonnegative integer for spray_power()

Value

These functions return a two-element list which is coerced to an object of class spray by function spraymaker()

Note

These functions aren’t really designed for the end-user.

Function spray_equality() cannot simply check for equality of $value because the order of the index rows is not specified in a spray object. Function spray_crush() has been removed as it is redundant.

Author(s)

Robin K. S. Hankin
See Also
spraymaker.spray

spray_missing_accessor

Description
Discussion about the difficulties of implementing idiom like $S[1,5]$ in the package

Usage
spray_missing_accessor(S, dots)

Arguments
S Object of class spray
dots further arguments

Details
File inst/missing_accessor.txt presents an extended discussion of the difficulties of implementing idiom like $S[1,5]$ in the package.

Author(s)
Robin K. S. Hankin

spraycross Cross product for spray objects

Description
Provides a natural cross product for spray objects, useful for tensors and $k$-forms

Usage
spraycross(S, ...)
spraycross2(S1, S2)

Arguments
S, S1, S2, ... spray objects

Details
Tensor products for sprays. This is not an algebraic product of sprays interpreted as multivariate polynomials. The function is used in the stokes package. Function spraycross2() is a helper function that takes exactly two arguments. Function spraycross() is a more general function that takes any number of arguments.
Value

Returns a spray object

Author(s)

Robin K. S. Hankin

Examples

```r
a <- spray(matrix(1:4,2,2),c(2,5))
b <- spray(matrix(c(10,11,12,13),2,2),c(7,11))
a b
spraycross2(a,b)
spraycross2(b,a)
spraycross(a,b,b)
```

---

**subs**

Substitute values into a spray object

Description

Substitute values into a spray object, interpreted as a multivariate polynomial

Usage

```
subs(S, dims, x, drop=TRUE)
```

Arguments

- **S**: spray object
- **dims**: Integer or logical vector with entries corresponding to the dimensions to be substituted
- **x**: Numeric vector of values to be substituted
- **drop**: Boolean, with default `TRUE` meaning to return the `drop()` of the result, and `FALSE` meaning to return a spray object consistently

Note

It is much easier if argument `dims` is sorted into increasing order. If not, caveat emptor!

Author(s)

Robin K. S. Hankin

See Also

`process_dimensions`
summary.spray

Examples

(S <- spray(matrix(sample(0:3,60,replace=TRUE),nrow=12)))

subs(S, c(2,5), 1:2)

P <- homog(3,3)
subs(P, 1, 2)

summary.spray

Summaries of spray objects

Description

A summary method for spray objects, and a print method for summaries.

Usage

## S3 method for class 'spray'
summary(object, ...)
## S3 method for class 'summary.spray'
print(x, ...)

Arguments

object, x
Object of class spray
...
Further arguments, passed to head()

Details

A summary.spray object is summary of a spray object x: a list with first element being a summary() of the coefficients (which is a disord object), and the second being a spray object comprising a few selected index-coefficient pairs. The selection is done by head().

Note

The “representative selection” is implementation-specific, as it uses disordR::elements() to extract rows of the index matrix and coefficients.

Author(s)

Robin K. S. Hankin

Examples

a <- rspray()^2
a
summary(a)
summary(a, 2)

options(polyform=TRUE)
summary(a^4, 3)
options(polyform=FALSE) # restore default
Zap small values in a spray object

Description

Generic version of zapsmall()

Usage

zap(x, digits = getOption("digits"))
## S4 method for signature 'spray'
zapsmall(x, digits = getOption("digits"))

Arguments

- x: spray object
- digits: number of digits to retain

Details

Given a spray object, coefficients close to zero are ‘zapped’, i.e., replaced by ‘0’, using base::zapsmall(). Function zap() is an easily-typed alias; zapsmall() is the S4 generic.

Note, zap() actually changes the numeric value, it is not just a print method.

Author(s)

Robin K. S. Hankin

Examples

```r
(S <- spray(matrix(sample(1:50),ncol=2),10^-(1:25)))
zap(S)
S-zap(S) # print method will probably print zeros...
coeffs(S-zap(S)) # ...but they are nevertheless nonzero
```

The zero polynomial

Description

Test for the zero, or empty, polynomial

Usage

- zero(d)
- is.zero(x)
- is.empty(L)
Arguments

L, x  A two-element list of indices and values, possibly a spray object or numeric vector

d  Integer specifying dimensionality of the spray (the arity)

Details

Functions `is.empty()` and `is.zero()` are synonyms. If spray objects are interpreted as multivariate polynomials, “is.zero()” is more intuitive, if sprays are interpreted as sparse arrays, “is.empty()” is better (for me).

Passing a zero-row index matrix can have unexpected effects:

```r
dput(spray(matrix(0,0,5),9))
structure(list(structure(numeric(0), .Dim = c(0L, 5L)), numeric(0)), class = "spray")
```

Above, the index matrix has zero rows (and no elements) but the fact that it has five columns is retained. Arguably `spray()` should return an error here, as the number of rows of the index matrix should match the length of the coefficient vector and they do not: the index has zero rows and the coefficient vector has length 1 (although they match in the returned value). The returned spray object has no coefficients [specifically, numeric(0)]; this is consistent with the index matrix having zero rows.

Zero coefficients are discarded by the back end:

```r
> spray(matrix(1,1,5),0)
empty sparse array with 5 columns
> dput(spray(matrix(1,1,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
```

Above, the index matrix given to `spray()` has one row but the coefficient is a length-one vector with element zero. The resulting spray object has a NULL index matrix [because rows with zero coefficients are removed] and a NULL coefficient. It is also permissible to pass a a zero-row matrix:

```r
spray(matrix(0,0,5),0)
empty sparse array with 5 columns

dput(spray(matrix(0,0,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
```

In previous versions of the package, the index matrix in the returned spray object could be NULL under some circumstances. If so, the arity of the spray object is lost. It is probably worth noting that `spray()`, given a zero-row index matrix, loses a length one coefficients vector, but complains about a length-two coefficient vector:

```r
> dput(spray(matrix(0,0,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
> dput(spray(matrix(0,0,5),3))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
> dput(spray(matrix(0,0,5),1:2))
Error in is_valid_spray(L) : nrow(L[[1]]) == length(L[[2]]) is not TRUE
>
> identical(spray(matrix(0,0,5),0),spray(matrix(0,0,5),3))
[1] TRUE
**Examples**

(a <- lone(1,3))

is.zero(a-a)  # should be TRUE

is.zero(zero(6))

x <- spray(t(0:1))
y <- spray(t(1:0))

is.zero((x+y)*(x-y)-(x^2-y^2))  # TRUE
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