Package ‘spsur’

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**Description**
A collection of functions to test and estimate Seemingly Unrelated Regression (usually called SUR) models, with spatial structure, by maximum likelihood and three-stage least squares. The package estimates the most common spatial specifications, that is, SUR with Spatial Lag of X regressors (called SUR-SLX), SUR with Spatial Lag Model (called SUR-SLM), SUR with Spatial Error Model (called SUR-SEM), SUR with Spatial Durbin Model (called SUR-SDM), SUR with Spatial Durbin Error Model (called SUR-SDEM), SUR with Spatial Autoregressive terms and Spatial Autoregressive Disturbances (called SUR-SARAR), SUR-SARAR with Spatial Lag of X regressors (called SUR-GNM) and SUR with Spatially Independent Model (called SUR-SIM). The methodology of these models can be found in next references:


**LazyData**
true

**Encoding**
UTF-8

**License**
GPL-3

**Depends**
R (>= 4.1), methods (>= 4.1), stats (>= 4.1)

**Imports**
Formula (>= 1.2-4), ggplot2 (>= 3.3.6), gmodels (>= 2.18.1), gridExtra (>= 2.3), MASS (>= 7.3-56), Matrix (>= 1.4-1), minqa (>= 1.2.4), numDeriv (>= 2016.8-1.1), Rdpack (>= 2.4), rlang (>= 1.0.4), sparseMVN (>= 0.2.2), spatialreg (>= 1.2-5), spdep (>= 1.2-5), sphet(>= 2.0)

**Suggests**
dplyr (>= 1.0.9), knitr (>= 1.39), sf (>= 1.0-8)

**VignetteBuilder**
knitr

**RoxygenNote**
7.2.1
spsur-package

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spsur-package Spatial Seemingly Unrelated Regression Models.

Description

spsur offers the user a collection of functions to estimate Spatial Seemingly Unrelated Regression (SUR) models by maximum likelihood or three-stage least squares, using spatial instrumental variables. Moreover, spsur obtains a collection of misspecification tests for omitted or wrongly specified spatial structure. The user will find spatial models more popular in applied research such as the SUR-SLX, SUR-SLM, SUR-SEM, SUR-SDM, SUR-SDEM SUR-SARAR and SUR-GNM plus the spatially independent SUR, or SUR-SIM.
Details

Some functionalities that have been included in spsur package are:

1. Testing for spatial effects

The function `lmtestspsur` provides a collection of Lagrange Multipliers, LM, for testing different forms of spatial dependence in SUR models. They are extended versions of the well-known LM tests for omitted lags of the explained variable in the right hand side of the equation, LM-SLM, the LM tests for omitted spatial errors, LM-SEM, the joint test of omitted spatial lags and spatial errors, LM-SARAR, and the robust version of the first two Lagrange Multipliers, LM*-SLM and LM*-SEM.

These tests can be applied to models always with a SUR nature. Roughly, we may distinguish two situations:

- Datasets with a single equation $G=1$, for different time periods $Tm>1$ and a certain number of spatial units in the cross-sectional dimension, $N$. This is what we call spatial panel datasets. In this case, the SUR structure appears in form of (intra) serial dependence in the errors of each spatial unit.
- Datasets with a several equations $G>1$, different time periods $Tm>1$ and a certain number of spatial units, $N$. The SUR structure appears, as usual, because the errors of the spatial units for different equations are contemporaneously correlated.

2. Estimation of the Spatial SUR models

As indicated above, spsur package may work with a list of different spatial specifications. They are the following:

- **SUR-SIM**: SUR model without spatial effects
  \[
  y_{tg} = X_{tg} \beta_g + \epsilon_{tg}
  \]

- **SUR-SLX**: SUR model with spatial lags of the regressors
  \[
  y_{tg} = X_{tg} \beta_g + WX_{tg} \theta_g + \epsilon_{tg}
  \]

- **SUR-SLM**: SUR model with spatial lags of the endogenous
  \[
  y_{tg} = \rho_g Wy_{tg} + X_{tg} \beta_g + \epsilon_{tg}
  \]

- **SUR-SEM**: SUR model with spatial errors
  \[
  y_{tg} = X_{tg} \beta_g + u_{tg}
  
  u_{tg} = \lambda_g Wu_{tg} + \epsilon_{tg}
  \]

- **SUR-SDM**: SUR model with spatial lags of the endogenous variable and of the regressors or Spatial Durbin model
  \[
  y_{tg} = \rho_g Wy_{tg} + X_{tg} \beta_g + WX_{tg} \theta_g + \epsilon_{tg}
  \]
• **SUR-SDEM**: SUR model with spatial errors and spatial lags of the endogenous variable and of the regressors

\[ y_{tg} = X_{tg}\beta_g + WX_{tg}\theta_g + u_{tg} \]

\[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

• **SUR-SARAR**: Spatial lag model with spatial errors

\[ y_{tg} = \rho_g Wy_{tg} + X_{tg}\beta_g + u_{tg} \]

\[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

• **SUR-GNM**: SUR model with spatial lags of the explained variables, regressors and spatial errors

\[ y_{tg} = \rho_g Wy_{tg} + X_{tg}\beta_g + WX_{tg}\theta_g + u_{tg} \]

\[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

where \( y_{tg}, u_{tg} \) and \( \epsilon_{tg} \) are \((N\times1)\) vectors; \( X_{tg} \) is a matrix of regressors of order \((N\times P)\); \( \rho_g \) and \( \lambda_g \) are parameters of spatial dependence and \( W \) is the \((N\times N)\) spatial weighting matrix.

These specifications can be estimated by maximum-likelihood methods, using the function `spsurml`. Moreover, the models that include spatial lags of the explained variables in the right hand side of the equations, and the errors are assumed to be spatially incorrelated (namely, the SUR-SLM and the SUR-SDM), can also be estimated using three-stage least-squares, `spsur3sls`, using spatial instrumental variable to correct for the problem of endogeneity present in these cases.

### 3. Diagnostic tests

Testing for inconsistencies or misspecifications in the results of an estimated (SUR) model should be a primary task for the user. `spsur` focuses, specifically, on two main question such as omitted or wrongly specified spatial structure and the existence of structural breaks or relevant restrictions in the parameters of the model. In this sense, the user will find:

1. **Marginal tests**

   The Marginal Multipliers test for omitted or wrongly specified spatial structure in the equations. They are routinely part of the output of the maximum-likelihood estimation, shown by `spsurml`. In particular, the LM(\(\rho|\lambda\)) tests for omitted spatial lags in a model specified with spatial errors (SUR-SEM; SUR-SDEM). The LM(\(\lambda|\rho\)) tests for omitted spatial error in a model specified with spatial lags of the explained variable (SUR-SLM; SUR-SDM).

2. **Coefficients stability tests**

   `spsur` includes two functions designed to test for linear restrictions on the \(\beta\) coefficients of the models and on the spatial coefficients (\(\rho_s\) and \(\lambda_s\) terms). The function for the first case is `wald_betas` and `wald_deltas` that of the second case. The user has ample flexibility to define different forms of linear restrictions, so that it is possible, for example, to test for their time constancy to identify structural breaks.
4. Marginal effects

In recent years, since the publication of LeSage and Pace (2009), it has become popular in spatial econometrics to evaluate the multiplier effects that a change in the value of a regressor, in a point in the space, has on the explained variable. `spsur` includes a function, `impacts`, that computes these effects. Specifically, `impacts` obtains the average, over the \( N \) spatial units and \( Tm \) time periods, of such a change on the contemporaneous value of the explained variable located in the same point as the modified variable. This is the so-called Average Direct effect. The Average Indirect effect measure the proportion of the impact that spills-over to other locations. The sum of the two effects is the Average Total effect.

These estimates are complemented with a measure of statistical significance, following the randomization approach suggested by LeSage and Pace (2009).

5. Additional functionalities

A particular feature of `spsur` is that the package allows to obtain simulated datasets with a SUR nature and the spatial structure decided by the user. This is the purpose of the function `dgp_spsur`. The function can be inserted into a more general code to solve, for example, Monte Carlo studies related to these type of models or, simply, to show some of the stylized characteristics of a SUR model with certain spatial structure.

Datasets

`spsur` includes three different datasets: `spc`, NCOVR and `spain.covid`. These sets are used to illustrate the capabilities of different functions. Briefly, their main characteristics are the following

- The `spc` dataset (Spatial Phillips-Curve) is a classical dataset taken from Anselin (1988, p. 203), of small dimensions.
- The `NCOVR` dataset comprises Homicides and a list of selected socio-economic variables for continental U.S. counties in four decennial census years: 1960, 1970, 1980 and 1990. It is freely available from https://geodacenter.github.io/data-and-lab/ncovr/. `NCOVR` is a typical spatial panel dataset (\( G=1 \)).
- The `spain.covid` dataset comprises Within and Exit mobility index together with the weekly incidence COVID-19 at Spain provinces from February 21 to May 21 2020. https://www.mitma.gob.es/ministerio/covid-19/evolucion-movilidad-big-data

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References


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**dgp_spsur**

*Generation of a random dataset with a spatial SUR structure.*

**Description**

The purpose of the function `dgp_spsur` is to generate a random dataset with the dimensions and spatial structure decided by the user. This function may be useful in pure simulation experiments or with the aim of showing specific properties and characteristics of a spatial SUR dataset and inferential procedures related to them.

The user of `dgp_spsur` should think in terms of a Monte Carlo experiment. The arguments of the function specify the dimensions of the dataset to be generated, the spatial mechanism underlying the data, the intensity of the SUR structure among the equations and the values of the parameters to be used to obtain the simulated data, which includes the error terms, the regressors and the explained variables.

**Usage**

```r
dgp_spsur(Sigma, Tm = 1, G, N, Betas, Thetas = NULL, 
  rho = NULL, lambda = NULL, p = NULL, listw = NULL, 
  X = NULL, type = "matrix", pdfU = "nvrnorm", 
  pdfX = "nvrnorm")
```

**Arguments**

- **Sigma**
  - Covariance matrix between the `G` equations of the SUR model. This matrix should be definite positive and the user must check for that.
- **Tm**
  - Number of time periods. Default = 1
**dgp_spsur**

- **G** Number of equations.
- **N** Number of cross-section or spatial units
- **Betas** A row vector of order $(1 \times P)$ showing the values for the beta coefficients. The first $P_1$ terms correspond to the first equation (where the first element is the intercept), the second $P_2$ terms to the coefficients of the second equation and so on.
- **Thetas** Values for the $\theta$ coefficients in the $G$ equations of the model, when the type of spatial SUR model to be simulated is a "slx", "sdm" or "sdem". Thetas is a row vector of order $1 \times P_{Thetas}$, where $P_{Thetas} = p - G$; let us note that the intercept cannot appear among the spatial lags of the regressors. The first $1 \times K_{Thetas1}$ terms correspond to the first equation, the second $1 \times P_{Thetas2}$ terms correspond to the second equation, and so on. Default = NULL.
- **rho** Values of the coefficients $\rho_g; g = 1, 2, ..., G$ related to the spatial lag of the explained variable of the $g$-th equation. If $\rho$ is an scalar and there are $G$ equations in the model, the same value will be used for all the equations. If $\rho$ is a row vector, of order $(1 \times G)$, the function dgp_spsur will use these values, one for each equation. Default = NULL.
- **lambda** Values of the coefficients $\lambda_g; g = 1, 2, ..., G$ related to the spatial lag of the errors in the $G$ equations. If $\lambda$ is an scalar and there are $G$ equations in the model, the same value will be used for all the equations. If $\lambda$ is a row vector, of order $(1 \times G)$, the function dgp_spsur will use these values, one for each equation of the spatial errors. Default = NULL.
- **p** Number of regressors by equation, including the intercept. $p$ can be a row vector of order $(1 \times G)$, if the number of regressors is not the same for all the equations, or a scalar, if the $G$ equations have the same number of regressors.
- **listw** A listw object created for example by nb2listw from spatialreg package; if nb2listw not given, set to the same spatial weights as the listw argument. It can also be a spatial weighting matrix of order $(N \times N)$ instead of a listw object. Default = NULL.
- **X** This argument tells the function dgp_spsur which $X$ matrix should be used to generate the SUR dataset. If $X$ is different from NULL, dgp_spsur will upload the $X$ matrix selected in this argument. Note that the $X$ must be consistent with the dimensions of the model. If $X$ is NULL, dgp_spsur will generate the desired matrix of regressors from a multivariate Normal distribution with mean value zero and identity $(P \times P)$ covariance matrix. As an alternative, the user may change this probability distribution function to the uniform case, $U(0, 1)$, through the argument pdfX. Default = NULL.
- **type** Selection of the type of output. The alternatives are matrix, df, panel, all. Default matrix
- **pdfU** Multivariate probability distribution function, Mpdf, from which the values of the error terms will be drawn. The covariance matrix is the $\Sigma$ matrix specified by the user in the argument. Two alternatives "lognvrnorm", "nvrnorm". Default "nvrnorm".

**Sigma**. The function dgp_spsur provides two Mpdf, the multivariate Normal, which is the default, and the log-Normal distribution function which means
just exponentiate the sampling drawn from a $N(0, \Sigma)$ distribution. Default = "nvrnorm".

pdfX Multivariate probability distribution function (Mpdf), from which the values of the regressors will be drawn. The regressors are assumed to be independent.

dgp_spsur provides two Mpdf, the multivariate Normal, which is the default, and the uniform in the interval $U[0, 1]$, using the dunif function. dunif, from the stats package. Two alternatives "nvrunif", "nvrnorm". Default "nvrnorm".

Details

The purpose of the function dgp_spsur is to generate random datasets, of a SUR nature, with the spatial structure decided by the user. The function requires certain information to be supplied externally because, in fact, dgp_spsur constitutes a Data Generation Process, DGP. The following aspects should be addressed:

- The user must define the dimensions of the dataset, that is, number of equations, $G$, number of time periods, $T_m$, and number of cross-sectional units, $N$.

- The user must choose the type of spatial structure desired for the model from among the list of candidates of "sim", "slx", "slm", "sem", "sdm", "sdem" or "sarar". The default is the "sim" specification which does not have spatial structure. The decision is made implicitly, just omitting the specification of the spatial parameters which are not involved in the model (i.e., in a "slm" there are no $\lambda$ parameters but appear $\rho$ parameters; in a "sdem" model there are $\lambda$ and $\theta$ parameters but no $\rho$ coefficients).

- If the user needs a model with spatial structure, a $(N \times N)$ weighting matrix, $W$, should be chosen.

- The next step builds the equations of the SUR model. In this case, the user must specify the number of regressors that intervene in each equation and the coefficients, $\beta$ parameters, associated with each regressor. The first question is solved through the argument $p$ which, if a scalar, indicates that the same number of regressors should appear in all the equations of the model; if the user seeks for a model with different number of regressors in the $G$ equations, the argument $p$ must be a $(1 \times G)$ row vector with the required information. It must be remembered that dgp_spsur assumes that an intercept appears in all equations of the model.

The second part of the problem posited above is solved through the argument Betas, which is a row vector of order $(1 \times p)$ with the information required for this set of coefficients.

- The user must specify, also, the values of the spatial parameters corresponding to the chosen specification; we are referring to the $\rho_g$, $\lambda_g$ and $\theta_g$, for $g = 1, ..., G$, and $K_g$ parameters. This is done thought the arguments rho, lambda and theta. The first two, rho and lambda, work as $K$: if they are scalar, the same value will be used in the $G$ equations of the SUR model; if they are $(1 \times G)$ row vectors, a different value will be assigned for each equation.

Moreover, Theta works like the argument Betas. The user must define a row vector of order $1 \times PTheta$ showing these values. It is worth to remember that in no case the intercept will appear among the lagged regressors.

- With the argument type the user take the decision of the output format. See Value section.

- Finally, the user must decide which values of the regressors and of the error terms are to be used in the simulation. The regressors can be uploaded from an external matrix generated previously by the user. This is the argument $X$. It is the responsibility of the user to check that the dimensions of the external matrix are consistent with the dataset required for the SUR
model. A second possibility implies the regressors to be generated randomly by the function `dgp_spsur`. In this case, the user must select the probability distribution function from which the corresponding data (of the regressors and the error terms) are to be drawn.

dgp_spsur provides two multivariate distribution functions, namely, the Normal and the log-Normal for the errors (the second should be taken as a clear departure from the standard assumption of normality). In both cases, random matrices of order \(TmNsG\) are obtained from a multivariate normal distribution, with a mean value of zero and the covariance matrix specified in the argument \(Sigma\); then, this matrix is exponentiated for the log-Normal case. Roughly, the same procedure applies for drawing the values of the regressor. There are two distribution functions available, the normal and the uniform in the interval \(U[0,1]\); the regressors are always independent.

Value

The default output ("matrix") is a list with a vector \(Y\) of order \(TmNGx1\) with the values generated for the explained variable in the \(G\) equations of the SUR and a matrix \(XX\) of order \((TmNGx-sum(p))\), with the values generated for the regressors of the SUR, including an intercept for each equation.

In case of \(Tm = 1\) or \(G = 1\) several alternatives output can be select:

- If the user select `type = "df"` the output is a data frame where each column is a variable.
- If the user select `type = "panel"` the output is a data frame in panel format including two factors. The first factor point out the observation of the individual and the second the equation for different \(Tm\) or \(G\).
- Finally, if `type = "all"` is select the output is a list including all alternatives format.

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References


See Also

`spsurml, spsur3sls, spsurtim`
Examples

```r
## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_dgp_spsur, package="spsur")

### PANEL DATA (Tm = 1 or G = 1) ###

#### Example 1: DGP SLM model. G equations

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impactspsur

Direct, indirect and total effects estimated for a spatial SUR model

Description

This function is a wrapper for \texttt{impacts} method used in \texttt{spatialreg} package. Nevertheless, in this case the same method is used for both \texttt{lagsarlm} and \texttt{lmSLX} objects. For details of implementation, see the documentation of \texttt{impacts} function in \texttt{spatialreg} package.

The function obtains the multiplier effects, on the explained variable, of a change in a regressor for the model that has been estimated. For reasons given below, this function only applies to models with an autoregressive structure ("slm", "sdm", "sarar" and "gnm") or with spatial lags of the regressors ("slx", "sdem").

The measurement of the multiplier effects is a bit more complicated than in a pure time series context because, due to the spatial structure of the model, part of the impacts spills over non uniformly over the space. Using the notation introduced by LeSage and Pace (2009) we distinguish between:

- **Average Direct effects**: The average over the \(N\) spatial units and \(Tm\) time periods of the effect of a unitary change in the value of an explanatory variable on the contemporaneous value of the corresponding explained variable, located in the same point of the intervened regressor. This calculus is solved for all the regressors that appear in the \(G\) equations of the model.

- **Average Indirect effects**: The average over the \(N\) spatial units and \(Tm\) time periods of the effects of a unitary change in the value of an explanatory variable on the contemporaneous value of the corresponding explained variable, located in a different spatial unit that that of the intervened regressor. This calculus is solved for all the regressors that appear in the \(G\) equations of the model.

- **Average total effects**: The sum of Direct and Indirect effects.

The information on the three estimated effects is supplement with an indirect measure of statistical significance obtained from the randomization approach introduced in LeSage and Pace (2009).

Usage

```r
impactspsur(obj, ..., tr = NULL, R = NULL, listw = NULL, 
    evalues = NULL, tol = 1e-06, 
    empirical = FALSE, Q = NULL)
```
Arguments

**obj**
An `spsur` object created by `spsurml, spsur3sls` or `spsurtime`.

... Arguments passed through to methods in the `coda` package

**tr**
A vector of traces of powers of the spatial weights matrix created using `trW`, for approximate impact measures; if not given, `listw` must be given for exact measures (for small to moderate spatial weights matrices); the traces must be for the same spatial weights as were used in fitting the spatial regression, and must be row-standardised.

**R**
If given, simulations are used to compute distributions for the impact measures, returned as `mcmc` objects; the objects are used for convenience but are not output by an MCMC process.

**listw**
If `tr` is not given, a spatial weights object as created by `nb2listw`; they must be the same spatial weights as were used in fitting the spatial regression, but do not have to be row-standardised.

**evals**
vector of eigenvalues of spatial weights matrix for impacts calculations

**tol**
Argument passed to `mvnorm`: tolerance (relative to largest variance) for numerical lack of positive-definiteness in the coefficient covariance matrix

**empirical**
Argument passed to `mvnorm` (default `FALSE`): if true, the coefficients and their covariance matrix specify the empirical not population mean and covariance matrix.

**Q**
default `NULL`, else an integer number of cumulative power series impacts to calculate if `tr` is given.

Details

LeSage and Pace (2009) adapt the classical notion of 'economic multiplier' to the problem of measuring the impact that a unitary change in the value of a regressor, produced in a certain point in space, has on the explained variable. The question is interesting because, due to the spatial structure of the model, the impacts of such change spill non uniformly over the space. In fact, the reaction of the explained variable depends on its relative location in relation to the point of intervention.

To simplify matters, LeSage and Pace (2009) propose to obtain aggregated multipliers for each regressor, just averaging the $N^2$ impacts that results from intervening the value of each regressor on each of the $N$ points in Space, on the explained variable, measured also in each of the $N$ points in space. This aggregated average is the so-called **Total effect**.

Part of this impact will be absorbed by the explained variable located in the same point of the regressor whose value has been changed (for example, the k-th regresor in the g-th equation, in the n-th spatial unit) or, in other words, we expect that $\frac{dy_{tn}}{dx_{ktn}}_{ne0}$. The aggregated average for the $N$ points in space ($n=1,2,...,N$) and $Tm$ time periods is the so-called **Direct effect**. The difference between the **Total effect** and the **Direct effect** measures the portion of the impact on the explained variable that leaks to other points in space, $\frac{dy_{tn}}{dx_{ktn}}_{formem}$; this is the **Indirect effect**.

`impacts` obtains the three multipliers together with an indirect measure of statistical significance, according to the randomization approach described in Lesage and Pace (2009). Briefly, they suggest to obtain a sequence of `nsim` random matrices of order $(NTmxG)$ from a multivariate normal distribution $N(0; \Sigma)$, being $\Sigma$ the estimated covariance matrix of the $G$ equations in the SUR model. These random matrices, combined with the observed values of the regressors and the
estimated values of the parameters of the corresponding spatial SUR model, are used to obtain simulated values of the explained variables. Then, for each one of the \( n_{sim} \) experiments, the SUR model is estimated, and the effects are evaluated. The function \texttt{impacts} obtains the standard deviations of the \( n_{sim} \) estimated effects in the randomization procedure, which are used to test the significance of the estimated effects for the original data.

Finally, let us note that this is a SUR model where the \( G \) equations are connected only through the error terms. This means that if we intervene a regressor in equation \( g \), in any point is space, only the explained variable of the same equation \( g \) should react. The impacts do not spill over equations. Moreover, the impact of a regressor, intervened in the spatial unit \( n \), will cross the borders of this spatial unit only if in the right hand side of the equation there are spatial lags of the explained variables or of the regressors. In other words, the \textit{Indirect effect} is zero for the "sim" and "sem" models. \texttt{impacts} produces no output for these two models. Lastly, it is clear that all the impacts are contemporaneous because the equations in the SUR model have no time dynamics.

**Value**

A list of \( G \) objects either of class \texttt{lagImpact} or \texttt{WXImpact}.

For each of the \( G \) objects of the list, if no simulation is carried out the object returned is a list with:

- \texttt{direct} numeric vector
- \texttt{indirect} numeric vector
- \texttt{total} numeric vector

and a matching \texttt{Qres} list attribute if \( Q \) was given.

On the other hand, for each of the \( G \) objects of the list, if simulation is carried out the object returned is a list with:

- \texttt{res} a list with three components as for the non-simulation case, with a matching \texttt{Qres} list attribute if \( Q \) was given.
- \texttt{sres} a list with three \texttt{mcmc} matrices, for the direct, indirect and total impacts with a matching \texttt{Qmcmc} list attribute if \( Q \) was given.

**Author(s)**

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**References**


See Also

`impacts`, `spsurml`, `spsur3sls`

Examples

```r
## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_impactspsur, package="spsur")

# Pure Cross Sectional Data (G>1; Tm=1)

### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
## For SLM, SDM and SARAR models the output is a list of "lagImpact" objects
## See spatialreg::impacts for details.
spcsur_slm <- spsurml(formula = Tformula, data = spc,
type = "slm", listw = lwspc)
summary(spcsur_slm)
impacts_slm <- impactspsur(spcsur_slm, listw = lwspc, R = 1000)
## Impacts equation 1
summary(impacts_slm[[1]], zstats = TRUE, short = TRUE)
## Impacts equation 2
summary(impacts_slm[[2]], zstats = TRUE, short = TRUE)
## For SLX and SDEM models the output is a list of "WXImpact" objects
## See spatialreg::impacts for details.
## A SUR-SLX model
spcsur_slx <- spsurml(formula = Tformula, data = spc,
type = "slx", listw = lwspc)
summary(spcsur_slx)
impacts_slx <- impactspsur(spcsur_slx, listw = lwspc)
## A SUR-SDM model
spcsur_sdm <- spsurml(formula = Tformula, data = spc,
```
Testing for the presence of spatial effects in Seemingly Unrelated Regressions
The function `spsurml` reports a collection of Lagrange Multipliers designed to test for the presence of different forms of spatial dependence in a SUR model of the "sim" type. That is, the approach of this function is from 'specific to general'. As said, the model of the null hypothesis is the "sim" model whereas the model of the alternative depends on the effect whose omission we want to test.

The collection of Lagrange Multipliers obtained by `lmtestspsur` are standard in the literature and take into account the multivariate nature of the SUR model. As a limitation, note that each Multiplier tests for the omission of the same spatial effects in all the cross-sections of the $G$ equations.

### Usage

```r
lmtestspsur(...)
```

#### S3 method for class 'formula'

```r
lmtestspsur(
  formula,
  data,
  listw,
  na.action,
  time = NULL,
  Tm = 1,
  zero.policy = NULL,
  R = NULL,
  b = NULL,
  ...
)
```

#### Default S3 method:

```r
lmtestspsur(Y, X, G, N, Tm = 1, listw, p, R = NULL, b = NULL, ...)
```

### Arguments

- `...`: further arguments passed to the method.
- `formula`: An object type `Formula` similar to objects created with the package `Formula` describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
- `data`: An object of class `data.frame` or a matrix.
- `listw`: A listw object created for example by `nb2listw` from `spatialreg` package; if `nb2listw` not given, set to the same spatial weights as the listw argument. It can also be a spatial weighting matrix of order $(N \times N)$ instead of a listw object. Default = NULL.
- `na.action`: A function (default options("na.action")), can also be `na.omit` or `na.exclude` with consequences for residuals and fitted values. It may be necessary to set `zero.policy` to `TRUE` because this subsetting may create no-neighbour observations.
- `time`: time index for the spatial panel SUR data.
Number of time periods.

Similar to the corresponding parameter of \texttt{lagsarlm} function in \texttt{spatialreg} package. If \texttt{TRUE} assign zero to the lagged value of zones without neighbours, if \texttt{FALSE} assign \texttt{NA} - causing \texttt{spsurml()} to terminate with an error. Default = \texttt{NULL}.

A row vector of order \((Ixp)\) with the set of \(r\) linear constraints on the \(beta\) parameters. The first restriction appears in the first \(p\) terms, the second restriction in the next \(p\) terms and so on. Default = \texttt{NULL}.

A column vector of order \((rx1)\) with the values of the linear restrictions on the \(beta\) parameters. Default = \texttt{NULL}.

A column vector of order \((NTmGx1)\), with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) cross-sectional/spatial units. The specification of \(Y\) is only necessary if not available a \texttt{Formula} and a data frame. Default = \texttt{NULL}.

A data matrix of order \((NTmGxp)\) with the observations of the regressors. The number of covariates in the SUR model is \(p = \text{sum}(p_g)\) where \(p_g\) is the number of regressors (including the intercept) in the \(g\)-th equation, \(g = 1,...,G\). The specification of “\(X\)” is only necessary if not available a \texttt{Formula} and a data frame. Default = \texttt{NULL}.

Number of equations.

Number of cross-section or spatial units

Number of regressors by equation, including the intercept. \(p\) can be a row vector of order \((I\times G)\), if the number of regressors is not the same for all the equations, or a scalar, if the \(G\) equations have the same number of regressors. The specification of \(p\) is only necessary if not available a \texttt{Formula} and a data frame.

\texttt{lmtestspsur} tests for the omission of spatial effects in the "sim" version of the SUR model:

\[
y_{tg} = X_{tg} \beta_g + u_{tg}
\]

\[
E[u_{tg}u'_{th}] = \sigma_{gh}I_N \quad E[u_{tg}u'_{sh}] = 0 \text{ if } t \neq s
\]

where \(y_{tg}\) and \(u_{tg}\) are \((Nx1)\) vectors, corresponding to the \(g\)-th equation and time period \(t\); \(X_{tg}\) is the matrix of exogenous variables, of order \((N\times p_g)\). Moreover, \(\beta_g\) is an unknown \((p_gx1)\) vector of coefficients and \(\sigma_{gh}I_N\) the covariance between equations \(g\) and \(h\), being \(\sigma_{gh}\) and scalar and \(I_N\) the identity matrix of order \(N\). The Lagrange Multipliers reported by this function are the followings:

- **LM-SUR-LAG**: Tests for the omission of a spatial lag of the explained variable in the right hand side of the "sim" equation. The model of the alternative is:

\[
y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + u_{tg}
\]

The null and alternative hypotheses are:

\(H_0 : \rho_g = 0 (\text{forall}g) \text{ vs } H_A : \rho_g \neq 0 (\text{exist}g)\)
• **LM-SUR-ERR**: Tests for the omission of spatial dependence in the equation of the errors of the "sim" model. The model of the alternative is:

\[ y_{tg} = X_{tg} \beta_g + u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

The null and alternative hypotheses are:

\[ H_0 : \lambda_g = 0 \text{(forall} g \text{)} \text{ vs } H_A : \lambda_g \neq 0 \text{(exist} g \text{)} \]

• **LM-SUR-SARAR**: Tests for the simultaneous omission of a spatial lag of the explained variable in the right hand side of the "sim" equation and spatial dependence in the equation of the errors. The model of the alternative is:

\[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

The null and alternative hypotheses are:

\[ H_0 : \rho_g = \lambda_g = 0 \text{(forall} g \text{)} \text{ vs } H_A : \rho_g \neq 0 \text{or} \lambda_g \neq 0 \text{(exist} g \text{)} \]

• **LM*-SUR-SLM** and **LM*-SUR-SEM**: These two tests are the robustified version of the original, raw Multipliers, **LM-SUR-SLM** and **LM-SUR-SEM**, which can be severely oversized if the respective alternative hypothesis is misspecified (this would be the case if, for example, we are testing for omitted lags of the explained variable whereas the problem is that there is spatial dependence in the errors, or vice versa). The null and alternative hypotheses of both test are totally analogous to their twin non robust Multipliers.

**Value**

A list of htest objects each one including the Wald statistic, the corresponding p-value and the degrees of freedom.

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**References**


• Anselin, L. (1988) *Spatial econometrics: methods and models* Chap. 9 Dordrecht

See Also

spsurml, anova

Examples

#########################################################
######## CROSS SECTION DATA (G>1; Tm=1) # ############
#########################################################

rm(list = ls()) # Clean memory
data("spc")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
lwspc <- spdep::mat2listw(Wspc, style = "W")
lmtestspsur(formula = Tformula, data = spc, listw = lwspc)

## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_lmtestspsur, package="spsur")

#########################################################
######## PANEL DATA (G>1; Tm>1) ########
#########################################################

#### Example 2: Homicides & Socio-Economics (1960-90)
# Homicides and selected socio-economic characteristics for
# continental U.S. counties.
# https://geodacenter.github.io/data-and-lab/ncovr/
data(NCOVR, package="spsur")
nbncovr <- spdep::poly2nb(NCOVR.sf, queen = TRUE)
### Some regions with no links...
lwncovr <- spdep::nb2listw(nbncovr, style = "W", zero.policy = TRUE)
### With different number of exogenous variables in each equation
Tformula <- HR70 | HR80 | HR90 ~ PS70 + UE70 | PS80 + UE80 + RD80 |
            PS90 + UE90 + RD90 + PO90
lmtestspsur(formula = Tformula, data = NCOVR.sf, listw = lwncovr)

#########################################################
### PANEL DATA: TEMPORAL CORRELATIONS (G=1; Tm>1) ###
#########################################################

##### Example 3: NCOVR in panel data form
Year <- as.numeric(kronecker(c(1960,1970,1980,1990),
                             matrix(1,nrow = dim(NCOVR.sf)[1])))
HR <- c(NCOVR.sf$HR60,NCOVR.sf$HR70,NCOVR.sf$HR80,NCOVR.sf$HR90)
PS <- c(NCOVR.sf$PS60,NCOVR.sf$PS70,NCOVR.sf$PS80,NCOVR.sf$PS90)
UE <- c(NCOVR.sf$UE60,NCOVR.sf$UE70,NCOVR.sf$UE80,NCOVR.sf$UE90)
NCOVRpanel <- as.data.frame(cbind(Year,HR,PS,UE))
Tformula <- HR ~ PS + UE
lmtestspsur(formula = Tformula, data = NCOVRpanel, time = Year,
listw = lwncovr)

---

**lr_betas**

*Likelihood ratio for testing homogeneity constraints on beta coefficients of the SUR equations.*

**Description**

Function `lr_betas` obtains a Likelihood Ratio test, LR in what follows, with the purpose of testing if some of the $\beta$ coefficients in the $G$ equations of the SUR model are equal. This function has a straightforward application, especially when $G = 1$, to the case of testing for the existence of structural breaks in the $\beta$ parameters. The function can test for the homogeneity of only one coefficient, of a few of them or even the homogeneity of all the slope terms. The testing procedure implies, first, the estimation of both a constrained and a unconstrained model and, second, the comparison of the log-likelihoods to compute the LR statistics.

@usage lr_betas(obj, R, b)

**Usage**

`lr_betas(obj, R, b)`

**Arguments**

- **obj**
  An `spsur` object created by `spsurml`, `spsur3sls` or `spsurtime`.

- **R**
  A row vector of order $(1 \times Pr)$ showing the set of $r$ linear constraints on the $\beta$ parameters. The *first* restriction appears in the first $K$ terms in $R$, the *second* restriction in the next $K$ terms and so on.

- **b**
  A column vector of order $(rx1)$ with the values of the linear restrictions on the $\beta$ parameters.

**Value**

Object of `htest` including the LR statistic, the corresponding p-value, the degrees of freedom and the values of the sample estimates.

**Author(s)**

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Methods for class spsur

References


See Also

spsurml, spsurtime, wald_betas

Examples

```r
## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_lr_betas, package="spsur")

' # CROSS SECTION DATA (G>1; Tm=1) #

rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
### H0: equal beta for SMSA in both equations.
R <- matrix(c(0,0,0,1,0,0,0,-1), nrow=1)
b <- matrix(0, ncol=1)
spcsur.slm <- spsurml(formula = Tformula, data = spc,
                      type = "slm", listw = lwspc)
summary(spcsur.slm)
lr_betas(spcsur.slm, R = R, b = b)

### Estimate restricted SUR-SLM model
spcsur.slmr <- spsurml(formula = Tformula, data = spc,
                       type = "slm", listw = lwspc,
                       R = R, b = b)
summary(spcsur.slmr)
```
Description

The `anova()` function provides tables of fitted `spsur` models including information criteria (AIC and BIC), log-likelihood and degrees of freedom of each fitted model. The argument `lrtest` allows to perform LR tests between nested models. The `plot()` function allows the user to plot both beta and spatial coefficients for all equations of the `spsur` model. The argument `viewplot` is used to choose between interactive or non-interactive plots. The `print()` function is used to print short tables including the values of beta and spatial coefficients as well as p-values of significance test for each coefficient. This can be used as an alternative to `summary.spsur` when a brief output is needed. The rest of methods works in the usual way.

Usage

```r
## S3 method for class 'spsur'
anova(object, ..., lrtest = TRUE)

## S3 method for class 'spsur'
coef(object, ...)

## S3 method for class 'spsur'
fitted(object, ...)

## S3 method for class 'spsur'
logLik(object, ...)

## S3 method for class 'spsur'
residuals(object, ...)

## S3 method for class 'spsur'
vcov(object, ...)

## S3 method for class 'spsur'
print(x, digits = max(3L, getOption("digits") - 3L), ...)

## S3 method for class 'spsur'
plot(x, ci = 0.95, viewplot = TRUE, ...)
```

Arguments

- **object**: a `spsur` object created by `spsurml`, `spsur3sls` or `spsurtime`.
- **...**: further arguments passed to or from other methods.
- **lrtest**: logical value to compute likelihood ratio test for nested models in ‘anova’ method. Default = TRUE
- **x**: similar to object argument for `print()` and `plot` functions.
- **digits**: number of digits to show in printed tables. Default: max(3L, getOption("digits") - 3L).
- **ci**: confidence level for the intervals in ‘plot’ method. Default ci = 0.95
- **viewplot**: logical value to show interactively the plots. Default = TRUE
Examples

```r
rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
spcsur.sim <- spsurml(formula = Tformula, data = spc, type = "sim")
## Print Table
print(spcsur.sim)

spcsur.slm <- spsurml(formula = Tformula, data = spc, type = "slm",
                      listw = Wspc)
# ANOVA table and LR test for nested models:
anova(spcsur.sim, spcsur.slm)
## Plot spatial and beta coefficients
# Interactive plot
plot(spcsur.slm)
# Non-interactive plot
if (require(gridExtra)) {
  pl <- plot(spcsur.slm, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
                pl$pldeltas, nrow = 3)
}
```

NCOVR.sf

Homicides in U.S. counties

Description

Homicides and selected socio-economic characteristics for continental U.S. counties. Data for four

Usage

NCOVR.sf
Format

An spatial feature (sf) object with 3085 rows and 41 variables:

NAME  County coded as a name (factor)
STATE_NAME  state fips code (factor)
FIPS  state fips code (factor)
SOUTH dummy variable for Southern counties (South = 1)
HR60, HR70, HR80, HR90  homicide rate per 100,000 (1960, 1970, 1980, 1990)
HC60, HC70, HC80, HC90  homicide count, three year average centered on 1960, 1970, 1980, 1990
PO60, PO70, PO80, PO90  county population, 1960, 1970, 1980, 1990
RD60, RD70, RD80, RD90  resource deprivation 1960, 1970, 1980, 1990 (principal component, see Codebook for details)
PS60, PS70, PS80, PS90  population structure 1960, 1970, 1980, 1990 (principal component, see Codebook for details)
UE60, UE70, UE80, UE90  unemployment rate 1960, 1970, 1980, 1990
DV60, DV70, DV80, DV90  divorce rate 1960, 1970, 1980, 1990 (% males over 14 divorced)
MA60, MA70, MA80, MA90  median age 1960, 1970, 1980, 1990
FP59, FP69, FP79, FP89  % families below poverty 1960, 1970, 1980, 1990 (see Codebook for details)
geometry  Multipolygon geometry of the spatial feature object

Source


References


print.summary.spsur  

Print method for objects of class summary.spsur.

Description

Print method for objects of class summary.spsur.

Usage

## S3 method for class 'summary.spsur'
print(x, digits = max(3L, getOption("digits") - 3L), ...)
\textbf{spain.covid}

\section*{Arguments}
\begin{itemize}
\item \texttt{x} \hspace{2cm} object of class \texttt{summary.spsur}.
\item \texttt{digits} \hspace{2cm} number of digits to show in printed tables. Default: \texttt{max(3L, \texttt{getOption("digits") - 3L})}.
\item \ldots \hspace{2cm} further arguments passed to or from other methods.
\end{itemize}

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\section*{See Also}
\texttt{summary.spsur}.

\section*{Examples}
\begin{verbatim}
# See examples for \code{\link{spsurml}} or \\
# \code{\link{spsur3sls}} functions.
\end{verbatim}

\begin{tabular}{ll}
\hline
spain.covid & \textit{Within/Exit mobility index and incidence COVID-19 at Spain provinces} \\
\hline
\end{tabular}

\section*{Description}
Weekly within/exit mobility indices and COVID-19 incidence in Spain at provincial level. A total of 17 weeks from February 21 to May 21. Every week starts on a Friday and ends on the following Thursday. All travels are expressed with respect to the pre-COVID week of February 14th-20th, 2020 (week0). A value lower than 1 indicate a reduction of the mobility. Upper values than 1 indicate an increase in the mobility with respect to the reference week (week0).

\section*{Usage}
\texttt{spain.covid}

\section*{Format}
A data frame with 850 rows and 11 variables:
\begin{itemize}
\item \texttt{province} Province name coded as a factor
\item \texttt{indiv} Province coded as a number
\end{itemize}
time Number of week after pre-COVID (week0: February 14th-20th, 2020). time=1 (week1, February 21th-27th); time=2 (week2, February 28th-March 5th);...

Within Mobility index within of the province. See details for a formal definition

Exits Mobility index of exits of the province. See details for a formal definition

Emergence Dummy variable. 1 if the Emergence State ("Estado de Alarma") is active in the week "time" but economic activity of essential services are allowed. 0 in another case. The Emergence State was active in Spain from February 14th, 2020 to May 21th, 2020

EmergenceTotal Dummy variable. 1 if the Emergence State ("Estado de Alarma") is active in the week "time" and economic activity of essential services are not allowed. 0 in another case

Old65 Percentage of population aged 65 and older in the province

Density Inhabitants (in thousands) per km² in the province

Essential Percentage of firms in the province "indiv" with essential activities (food, health) over the total of firms in the province "indiv" in the province (e.g., food, health and some economic subsectors of industry and construction)

Incidence Weekly incidence in the week "time-1" in logs

Details

• Mobility

The mobility indices Within and Exits has been obtained as a ratio of the total number of weekly travels in reference to the total number of travels in the week of reference (week0).

In particular,

\[
\text{Within} = \frac{\text{Number of travels within the province 'indiv' in the week 'time'}}{\text{Number of travels within the province 'indiv' in the reference week (week0)}}
\]

\[
\text{Exits} = \frac{\text{Number of travels with origin in the province 'indiv' and arrival to another province in the week 'time'}}{\text{Number of travels with origin in the province 'indiv' and arrival to another province in the reference week (week0)}}
\]

A 'travel' is a displacement from an origin to a destination of at least 500m. A travel can have several stages. These stages of the same travel are calculated based on the duration of the intermediate stop. For example, if I move from origin A to destination B with a stop at a point C of long duration, it is considered two travels, but if the stop is short, it is considered a single travel. For example, I can go by train from Madrid to Alicante, and there takes a bus to Benidorm. The travel will be one (Madrid-Benidorm). If I do the same but the stop in Alicante is long (or an overnight stay, for example), two travels will be considered (Madrid-Alicante, and Alicante-Benidorm). Similarly, if I go by car from Madrid to Alicante and stop for 15 minutes to take a coffee, it is also considered only one travel and not two. The travels considered in this study are always from 500m, due to the limitation of source data that is based on mobile telephony and its antennas. But one travel can be 600 meters or 600km.

• Incidence

\[
\text{Incidence} = \log \left( \frac{\text{total diagnostic cases of COVID-19, PCR test in the week 'time' at the province 'indiv'}}{\text{total population in the province 'indiv'}} \right)
\]
Essential activities

Essential activities. Economic activities whose activities are essential for the population. By example, essential activities are healthcare, food supply, State security, media and communication, refuse collection, management and public transport, etc. Non essential activities are by example, restaurants, hotels, hairdressers, etc. A full list in the Spanish official bulletin

Source

The National Statistics Institute
Instituto de Salud Carlos III
https://cnecovid.isciii.es/covid19/

spain.covid.sf        Spain geometry

Description

A sf object with the Spanish geometry

Usage

spain.covid.sf

Format

A sf object with the Spanish geometry

PROVINCIA province name coded as a factor
ID_INE province coded as a number

spc        A classical Spatial Phillips-Curve

Description


Usage

spc
Format

A data frame with 25 rows and 10 variables:

COUNTY  County coded as a name.
WAGE83  Changes in wage rates for 1983.
UN83    Inverse unemployment rate in 1983.
NMR83   Net migration rate 1983.
SMSA    Dummy variable to identify counties defined as Standard Metropolitan Statistical Areas (SMSA = 1).
WAGE82  Changes in wage rates for 1982.
WAGE81  Changes in wage rates for 1981.
UN80    Inverse unemployment rate in 1980.
NMR80   Net migration rate 1983.
WAGE80  Changes in wage rates.
geometry geometry of sf object.

Source

Anselin (1988, p. 203-211)

References


Description

The function estimates spatial SUR models using three stages least squares, where the instruments are obtained from the spatial lags of the X variables, assumed to be exogenous. The number of equations, time periods and spatial units is not restricted. The user can choose between a Spatial Durbin Model or a Spatial Lag Model, as described below. The estimation procedure allows for the introduction of linear restrictions on the $\beta$ parameters associated to the regressors.

Usage

spsur3sls (formula = NULL, data = NULL, na.action,
R = NULL, b = NULL, listw = NULL,
zero.policy = NULL, X= NULL, Y = NULL, G = NULL,
N = NULL, Tm = NULL, p = NULL,
type = "slm", Durbin = NULL, maxlagW = NULL,
trace = TRUE)
Arguments

formula An object type \texttt{formula} similar to objects created with the package \texttt{Formula} describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.

data An object of class \texttt{data.frame} or a matrix.

na.action A function (default \texttt{options("na.action")}), can also be \texttt{na.omit} or \texttt{na.exclude} with consequences for residuals and fitted values. It may be necessary to set \texttt{zero.policy} to \texttt{TRUE} because this subsetting may create no-neighbour observations.

\( R \) A row vector of order \((1 \times p)\) with the set of \( r \) linear constraints on the \( beta \) parameters. The first restriction appears in the first \( p \) terms, the second restriction in the next \( p \) terms and so on. Default = \texttt{NULL}.

\( b \) A column vector of order \((r \times 1)\) with the values of the linear restrictions on the \( beta \) parameters. Default = \texttt{NULL}.

\texttt{listw} A list\( \texttt{w} \) object created for example by \texttt{nb2listw} from \texttt{spatialreg} package; if \texttt{nb2listw} not given, set to the same spatial weights as the \texttt{listw} argument. It can also be a spatial weighting matrix of order \((N \times N)\) instead of a \texttt{listw} object. Default = \texttt{NULL}.

\texttt{zero.policy} Similar to the corresponding parameter of \texttt{lagsarl} function in \texttt{spatialreg} package. If \texttt{TRUE} assign zero to the lagged value of zones without neighbours, if \texttt{FALSE} assign \texttt{NA} - causing \texttt{spsurml()} to terminate with an error. Default = \texttt{NULL}.

\( X \) A data matrix of order \((N T m G x p)\) with the observations of the regressors. The number of covariates in the SUR model is \( p = \sum p_g \) where \( p_g \) is the number of regressors (including the intercept) in the \( g \)-th equation, \( g = 1 \ldots G \). The specification of "\( X \)" is only necessary if not available a \texttt{Formula} and a data frame. Default = \texttt{NULL}.

\( Y \) A column vector of order \((N T m G x 1)\), with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) cross-sectional/spatial units. The specification of \( Y \) is only necessary if not available a \texttt{Formula} and a data frame. Default = \texttt{NULL}.

\( G \) Number of equations.

\( N \) Number of cross-section or spatial units.

\( Tm \) Number of time periods.

\( p \) Number of regressors by equation, including the intercept. \( p \) can be a row vector of order \((1 \times G)\), if the number of regressors is not the same for all the equations, or a scalar, if the \( G \) equations have the same number of regressors. The specification of \( p \) is only necessary if not available a \texttt{Formula} and a data frame.

\texttt{type} Type of spatial model, restricted to cases where lags of the explained variable appear in the right hand side of the equations. There are two possibilities: "slm" or "sdm". Default = "slm".

\texttt{Durbin} If a formula object and model is type "sdm" the subset of explanatory variables to lag for each equation.
maxlagW  Maximum spatial lag order of the regressors employed to produce spatial instruments for the spatial lags of the explained variables. Default = 2. Note that in case of type="sdm", the default value for maxlagW is set to 3 because the first lag of the regressors, WX_{tg}, cannot be used as spatial instruments.

trace  A logical value to show intermediate results during the estimation process. Default = TRUE.

Details

spsur3sls can be used to estimate two groups of spatial models:

• "slm": SUR model with spatial lags of the endogenous in the right hand side of the equations
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + \epsilon_{tg} \]

• "sdm": SUR model of the Spatial Durbin type
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + WX_{tg} \theta_g + \epsilon_{tg} \]

where \( y_{tg} \) and \( \epsilon_{tg} \) are \((N \times 1)\) vectors, corresponding to the \( g \)-th equation and time period \( t \); \( X_{tg} \) is the matrix of regressors, of order \((N \times p_g)\). Moreover, \( \rho_g \) is a spatial coefficient and \( W \) is a \((N \times N)\) spatial weighting matrix.

By default, the input of this function is an object created with Formula and a data frame. However, spsur3sls also allows for the direct specification of vector \( Y \) and matrix \( X \), with the explained variables and regressors respectively, as inputs (these terms may be the result, for example, of dgp_spsur).

spsur3sls is a Least-Squares procedure in three stages designed to circumvent the endogeneity problems due to the presence of spatial lags of the explained variable in the right hand side of the equations do the SUR. The instruments are produced internally by spsur3sls using a sequence of spatial lags of the \( X \) variables, which are assumed to be exogenous. The user must define the number of (spatial) instruments to be used in the procedure, through the argument maxlagW (i.e. maxlagW = 3). Then, the collection of instruments generated is \([WX_{tg}; WX_{tg}^2; WX_{tg}^3]\). In the case of a SDM, the first lag of the \( X \) matrix already is in the equation and cannot be used as instrument. In the example above, the list of instruments for a SDM model would be \([W^2 X_{tg}; W^3 X_{tg}]\).

The first stage of the procedure consists in the least squares of the \( Y \) variables on the set of instruments. From this estimation, the procedure retains the estimates of \( Y \) in the so-called \( Yls \) variables. In the second stage, the \( Y \) variables that appear in the right hand side of the equation are substituted by \( Yls \) and the SUR model is estimated by Least Squares. The third stage improves the estimates of the second stage through a Feasible Generalized Least Squares estimation of the parameters of the model, using the residuals of the second stage to estimate the Sigma matrix.

The arguments \( R \) and \( b \) allows to introduce linear restrictions on the beta coefficients of the \( G \) equations. spsur3sls, first, introduces the linear restrictions in the SUR model and builds, internally, the corresponding constrained SUR model. Then, the function estimates the restricted model which is shown in the output. The function does not compute the unconstrained model nor test for the linear restrictions. The user may ask for the unconstrained estimation using another spsurml estimation. Moreover, the function wald_betas obtains the Wald test of a set of linear restrictions for an object created previously by spsurml or spsur3sls.
Value

Object of spsur class with the output of the three-stages least-squares estimation of the specified spatial model. A list with:

call           Matched call.
type           Type of model specified.
Durbin          Value of Durbin argument.
coefficients    Estimated coefficients for the regressors.
deltas          Estimated spatial coefficients.
rest.se          Estimated standard errors for the estimates of \( \beta \) coefficients.
deltas.se       Estimated standard errors for the estimates of the spatial coefficients.
resvar          Estimated covariance matrix for the estimates of \( \beta \)'s and spatial coefficients.
R2              Coefficient of determination for each equation, obtained as the squared of the correlation coefficient between the corresponding explained variable and fitted values.
R2 pooled       Global coefficient of determination obtained for the set of the \( G \) equations. It is computed in the same way as uniequational \( R^2 \) but joining the dependent variable and fitted values in single vectors instead of one vector for each equation.
Sigma           Estimated covariance matrix for the residuals of the \( G \) equations.
residuals       Residuals of the model.
df.residuals     Degrees of freedom for the residuals.
fitted.values   Estimated values for the dependent variables.
G               Number of equations.
N               Number of cross-sections or spatial units.
Tm              Number of time periods.
p               Number of regressors by equation (including intercepts).
Y               If data is NULL, vector \( Y \) of the explained variables of the SUR model.
X               If data is NULL, matrix \( X \) of the regressors of the SUR model.
W               Spatial weighting matrix.
zero.policy     Logical value of zero.policy.
listw_style     Style of neighborhood matrix \( W \).

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References


See Also

`spsurML, stsls, wald_betass`

Examples

```r
#################################################
##### CLASSIC PANEL DATA (G=1; Tm>1) ################
#################################################

## A SUR model without spatial effects
rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA

## A SUR-SLM model (3SLS Estimation)
spcsur_slm_3sls <- spsur3sls(formula = Tformula, data = spc,
type = "slm", listw = lwspc)
summary(spcsur_slm_3sls)
print(spcsur_slm_3sls)

if (require(gridExtra)) {
  pl <- plot(spcsur_slm_3sls, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
               pl$pldeltas, nrow = 3)
}

## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_spsur3sls, package="spsur")

## A SUR-SDM model (3SLS Estimation)
spcsur_sdm_3sls <- spsur3sls(formula = Tformula, data = spc,
type = "sdm", listw = lwspc)
summary(spcsur_sdm_3sls)

if (require(gridExtra)) {
  pl <- plot(spcsur_sdm_3sls, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
               pl$pldeltas, nrow = 3)
}
rm(spcsur_sdm_3sls)
```
## A SUR-SDM model with different spatial lags in each equation

```r
TformulaD <- ~ UN83 + NMR83 + SMSA | UN80 + NMR80
spcsur_sdm2_3sls <- spsur3sls(formula = Tformula, data = spc,
                               type = "sdm", listw = lwspc,
                               Durbin = TformulaD)
```

```r
summary(spcsur_sdm2_3sls)
```

```r
if (require(gridExtra)) {
  pl <- plot(spcsur_sdm2_3sls, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
                pl$pldeltas, nrow = 3)
}
```

```r
rm(spcsur_sdm2_3sls)
```

### MULTI-DIMENSIONAL PANEL DATA (G>1; Tm>1) ###

#### Example 3: Homicides + Socio-Economics (1960-90)

# Homicides and selected socio-economic characteristics for continental
# U.S. counties.
# https://geodacenter.github.io/data-and-lab/ncovr/

```r
rm(list = ls()) # Clean memory
data(NCOVR, package = "spsur")
nbncovr <- spdep::poly2nb(NCOVR.sf, queen = TRUE)
# Some regions with no links...
lwncovr <- spdep::nb2listw(nbncovr, style = "W", zero.policy = TRUE)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
# A SUR-SLM model
NCOVRSUR_slm_3sls <- spsur3sls(formula = Tformula, data = NCOVR.sf,
                                type = "slm", zero.policy = TRUE,
                                listw = lwncovr, trace = FALSE)
```

```r
summary(NCOVRSUR_slm_3sls)
```

```r
if (require(gridExtra)) {
  pl <- plot(NCOVRSUR_slm_3sls, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
                pl$pldeltas, nrow = 3)
}
```

```r
rm(NCOVRSUR_slm_3sls)
```

---

**spsurgs3sls**

General Spatial 3SLS for systems of spatial equations.
Description

The function estimates spatial SUR models using general spatial three stages least squares. This is a system instrumental variable procedure which also include GMM estimation when there is spatial correlations in the errors. The procedure allows for additional endogenous regressors in addition to spatial lags of the dependent variable. It could be applied to "slm", "sdm", "sem" and "sarar" spatial models. Furthermore, for non-spatial models including endogenous regressors ("iv"), it could be used to estimate using instrumental variables and Feasible Generalized Least Squares.

Usage

spsurgs3sls(formula = NULL, data = NULL, na.action = options("na.action"), listw = NULL, zero.policy = NULL, type = "slm", Durbin = FALSE, endog = NULL, instruments = NULL, lag.instr = FALSE, initial.value = 0.2, het = FALSE, trace = TRUE)

Arguments

formula An object type Formula similar to objects created with the package Formula describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.

data An object of class data.frame or a matrix.

na.action A function (default options("na.action")), can also be na.omit or na.exclude with consequences for residuals and fitted values. It may be necessary to set zero.policy to TRUE because this subsetting may create no-neighbour observations.

listw A listw object created for example by nb2listw from spatialreg package; if nb2listw not given, set to the same spatial weights as the listw argument. It can also be a spatial weighting matrix of order (N x N) instead of a listw object. Default = NULL.

zero.policy Similar to the corresponding parameter of lagsarlm function in spatialreg package. (N x N) instead of a listw object. Default = NULL.

type Type of spatial model specification: "sim", "iv", "slm", "sem", "sdm" or "sarar". Default = "slm".

Durbin If a formula object and model is type "sdm" the subset of explanatory variables to lag for each equation.

endog Additional endogenous variables. Default NULL. If not NULL should be specified as a Formula with no dependent variable. Examples: ~ x1 | x2 (x1 endogeneous regressor for the first equation and x2 endogeneous regressor for the second equation) or ~ x1 | . (x1 endogenous regressor for the first equation and none endogenous regressors for the second equation)

instruments external instruments. Default NULL. If not NULL should be specified as a formula with no dependent variable in the same way than previous endog argument.

lag.instr should the external instruments be spatially lagged?
spsurg3sls

initial.value

- The initial value for rho. It can be either numeric (default is 0.2) or set to 'SAR', in which case the optimization will start from the estimated coefficient of a regression of the 2SLS residuals over their spatial lag (i.e. a spatial AR model).

het

- Default FALSE: if TRUE uses the methods developed for heteroskedasticity for each equation. Wrapper using spreg function.

trace

- A logical value to show intermediate results during the estimation process. Default = TRUE.

Details

spsurg3sls generalize the spreg function to multiequational spatial SUR models. The methodology to estimate spatial SUR models by Generalized 3SLS follows the steps outlined in Kelejian and Piras (pp. 304-305). The summary of the algorithm is the next one:

- Estimate each equation by 2SLS and obtain the estimated residuals \( \hat{u}_j \) for each equation.
- If the model includes a spatial lag for the errors, that is, it is a SEM/SARAR model, apply GMM to obtain the spatial parameters \( \hat{\lambda}_j \) for the residuals in each equation. In this case the spreg function is used as a wrapper for the GMM estimation. If the model does not include a spatial lag for the errors (that is, it is a "sim", "iv", "slm" or "sdm" model), then \( \hat{\lambda}_j = 0 \)
- Compute
  \[
  \hat{v}_j = \hat{u}_j - \hat{\lambda}_j W \hat{u}_j
  \]
  and the covariances
  \[
  \hat{\sigma}_{i,j} = N^{-1} \hat{v}_i \hat{v}_j
  \]
  Build \( \hat{\Sigma} = \{ \hat{\sigma}_{i,j} \} \)
- Compute
  \[
  y^*_j = y_j - \hat{\lambda}_j W y_j
  \]
  and
  \[
  X^*_j = X_j - \hat{\lambda}_j W X_j
  \]
  Compute
  \[
  \hat{X}^*_j = H_j (H_j^T H_j)^{-1} H_j^T X^*_j
  \]
  where \( H_j \) is the matrix including all the instruments and the exogenous regressors for each equation. That is, \( \hat{X}^*_j \) is the projection of \( X^*_j \) using the instruments matrix \( H_j \).
- Compute, in a multiequational way, the Feasible Generalized Least Squares estimation using the new variables \( y^*_j, \hat{X}^*_j \) and \( \hat{\Sigma} \). This is the 3sls step.

Value

Object of spsur class with the output of the three-stages least-squares estimation of the specified spatial model. A list with:

- call: Matched call.
- type: Type of model specified.
- Durbin: Value of Durbin argument.
- coefficients: Estimated coefficients for the regressors.
- deltas: Estimated spatial coefficients.
rest.se  Estimated standard errors for the estimates of $\beta$ coefficients.
deltas.se Estimated standard errors for the estimates of the spatial coefficients.
resvar Estimated covariance matrix for the estimates of $\beta$’s and spatial coefficients.
R2 Coefficient of determination for each equation, obtained as the squared of the correlation coefficient between
Sigma Estimated covariance matrix for the residuals of the $G$ equations.
residuals Residuals of the model.
df.residuals Degrees of freedom for the residuals.
fitted.values Estimated values for the dependent variables.
G Number of equations.
N Number of cross-sections or spatial units.
Tm Number of time periods.
p Number of regressors by equation (including intercepts).
Y If $data$ is $NULL$, vector $Y$ of the explained variables of the SUR model.
X If $data$ is $NULL$, matrix $X$ of the regressors of the SUR model.
W Spatial weighting matrix.
zero.policy Logical value of zero.policy.
listw_style Style of neighborhood matrix $W$.

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References


See Also

spreg, spsur3sls, stsls, spsurml
Examples

### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)

```r
rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
# No endogenous regressors
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
# Endogenous regressors and Instruments
Tformula2 <- WAGE83 | WAGE81 ~ NMR83 | NMR80
# Endogenous regressors: UN83, UN80
# Instrumental variable: SMSA

# A IV model with endogenous regressors only in first equation
spciv <- spsurgs3sls(formula = Tformula2, data = spc,
                      type = "iv", listw = lwspc,
                      endog = ~ UN83 | .,
                      instruments = ~ SMSA | .)
summary(spciv)
print(spciv)

# A SLM model with endogenous regressors
spcslm <- spsurgs3sls(formula = Tformula2, data = spc,
                      endog = ~ UN83 | .,
                      instruments = ~ SMSA | .,
                      type = "slm",
                      listw = lwspc)
summary(spcslm)
print(spcslm)

impacts_spcslm <- impactspsur(spcslm, listw = lwspc, R = 1000)
summary(impacts_spcslm[[1]], zstats = TRUE, short = TRUE)
summary(impacts_spcslm[[2]], zstats = TRUE, short = TRUE)

# A SDM model with endogenous regressors
spcsdm <- spsurgs3sls(formula = Tformula2, data = spc,
                      endog = ~ UN83 | UN80,
                      instruments = ~ SMSA | SMSA,
                      type = "sdm",
                      listw = lwspc,
                      Durbin = ~ NMR83 | NMR80)
summary(spcsdm)

# Durbin only in one equation
spcsdm2 <- spsurgs3sls(formula = Tformula2, data = spc,
                        endog = ~ UN83 | UN80,
                        instruments = ~ SMSA | SMSA,
                        type = "sdm",
                        listw = lwspc,
                        Durbin = ~ NMR83 | .)

summary(spcsdm2)

# A SEM model with endogenous regressors
spcsenm <- spsurgs3sls(formula = Tformula2, data = spc,
                       endog = ~ UN83 | UN80,
                       instruments = ~ SMSA | SMSA,
                       type = "sem",
                       listw = lwspc,
                       Durbin = ~ NMR83 | SMSA)
```

spsurgs3sls
spsurml

Maximum likelihood estimation of spatial SUR model.

Description
This function estimates spatial SUR models using maximum-likelihood methods. The number of equations, time periods and cross-sectional units is not restricted. The user can choose between different spatial specifications as described below. The estimation procedure allows for the introduction of linear restrictions on the $\beta$ parameters associated to the regressors.

Usage
spsurml(formula = NULL, data = NULL, na.action, listw = NULL, type = "sim", Durbin = NULL, method = "eigen", zero.policy = NULL, interval = NULL, trs = NULL, R = NULL, b = NULL, X = NULL, Y = NULL, G = NULL, N = NULL, Tm = NULL, p = NULL, control = list() )

Arguments

formula An object type Formula similar to objects created with the package Formula describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data An object of class data.frame or a matrix.
na.action A function (default options("na.action")), can also be na.omit or na.exclude with consequences for residuals and fitted values. It may be necessary to set zero.policy to TRUE because this subsetting may create no-neighbour observations.
spsurml

**listw**  A listw object created for example by `nb2listw` from `spatialreg` package; if `nb2listw` not given, set to the same spatial weights as the listw argument. It can also be a spatial weighting matrix of order (NxN) instead of a listw object. Default = NULL.

**type**  Type of spatial model specification: "sim", "slx", "slm", "sdm", "sdem", "sarar" or "gnm". Default = "sim".

**Durbin**  If a formula object and model is type "sdm", "sdem" or "slx" the subset of explanatory variables to lag for each equation.

**method**  Similar to the corresponding parameter of `lagsarlm` function in `spatialreg` package. "eigen" (default) - the Jacobian is computed as the product of (1 - rho*eigenvalue) using `eigenw`, and "spam" or "Matrix_J" for strictly symmetric weights lists of styles "B" and "C", or made symmetric by similarity (Ord, 1975, Appendix C) if possible for styles "W" and "S", using code from the spam or Matrix packages to calculate the determinant; "Matrix" and "spam_update" provide updating Cholesky decomposition methods; "LU" provides an alternative sparse matrix decomposition approach. In addition, there are "Chebyshev" and Monte Carlo "MC" approximate log-determinant methods; the Smirnov/Anselin (2009) trace approximation is available as "moments". Three methods: "SE_classic", "SE_whichMin", and "SE_interp" are provided experimentally, the first to attempt to emulate the behaviour of Spatial Econometrics toolbox ML fitting functions. All use grids of log determinant values, and the latter two attempt to ameliorate some features of "SE_classic".

**zero.policy**  Similar to the corresponding parameter of `lagsarlm` function in `spatialreg` package. If TRUE assign zero to the lagged value of zones without neighbours, if FALSE assign NA - causing `spsurml()` to terminate with an error. Default = NULL.

**interval**  Search interval for autoregressive parameter. Default = NULL.

**trs**  Similar to the corresponding parameter of `lagsarlm` function in `spatialreg` package. Default NULL, if given, a vector of powered spatial weights matrix traces output by `trW`.

**R**  A row vector of order (1xpr) with the set of r linear constraints on the beta parameters. The first restriction appears in the first p terms, the second restriction in the next p terms and so on. Default = NULL.

**b**  A column vector of order (rx1) with the values of the linear restrictions on the beta parameters. Default = NULL.

**X**  A data matrix of order (NTmGxp) with the observations of the regressors. The number of covariates in the SUR model is $p = \sum(p_g)$ where $p_g$ is the number of regressors (including the intercept) in the g-th equation, $g = 1,...,G$. The specification of "X" is only necessary if not available a `Formula` and a data frame. Default = NULL.

**Y**  A column vector of order (NTmgx1), with the observations of the explained variables. The ordering of the data must be (first) equation, (second) time dimension and (third) cross-sectional/spatial units. The specification of Y is only necessary if not available a `Formula` and a data frame. Default = NULL.

**G**  Number of equations.
N  Number of cross-section or spatial units
Tm Number of time periods.
p Number of regressors by equation, including the intercept. p can be a row vector of order (1xG), if the number of regressors is not the same for all the equations, or a scalar, if the G equations have the same number of regressors. The specification of p is only necessary if not available a Formula and a data frame.

control  list of additional arguments.

Details

The list of (spatial) models that can be estimated with the spsurml function are:

- "sim": SUR model with no spatial effects
  \[ y_{tg} = X_{tg} \beta_g + \epsilon_{tg} \]

- "slx": SUR model with spatial lags of the regressors
  \[ y_{tg} = X_{tg} \beta_g + WX_{tg} \theta_g + \epsilon_{tg} \]

- "slm": SUR model with spatial lags of the explained variables
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + \epsilon_{tg} \]

- "sem": SUR model with spatial errors
  \[ y_{tg} = X_{tg} \beta_g + u_{tg} \]
  \[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

- "sdm": SUR model of the Spatial Durbin type
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + WX_{tg} \theta_g + \epsilon_{tg} \]

- "sdem": SUR model with spatial lags of the regressors and spatial errors
  \[ y_{tg} = X_{tg} \beta_g + WX_{tg} \theta_g + u_{tg} \]
  \[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

- "sarar": SUR model with spatial lags of the explained variables and spatial errors
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + u_{tg} \]
  \[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

- "gnm": SUR model with spatial lags of the explained variables, regressors and spatial errors
  \[ y_{tg} = \rho_g W y_{tg} + X_{tg} \beta_g + WX_{tg} \theta_g + u_{tg} \]
  \[ u_{tg} = \lambda_g W u_{tg} + \epsilon_{tg} \]

Value

Object of spsur class with the output of the maximum-likelihood estimation of the specified spatial SUR model. A list with:
<table>
<thead>
<tr>
<th>call</th>
<th>Matched call.</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>Type of model specified.</td>
</tr>
<tr>
<td>method</td>
<td>Value of method argument to compute the Jacobian</td>
</tr>
<tr>
<td>Durbin</td>
<td>Value of Durbin argument.</td>
</tr>
<tr>
<td>coefficients</td>
<td>Estimated coefficients for the regressors.</td>
</tr>
<tr>
<td>deltas</td>
<td>Estimated spatial coefficients.</td>
</tr>
<tr>
<td>rest.se</td>
<td>Estimated standard errors for the estimates of beta.</td>
</tr>
<tr>
<td>deltas.se</td>
<td>Estimated standard errors for the estimates of the spatial coefficients (deltas).</td>
</tr>
<tr>
<td>resvar</td>
<td>Estimated covariance matrix for the estimates of beta's and spatial coefficients (deltas).</td>
</tr>
<tr>
<td>LL</td>
<td>Value of the likelihood function at the maximum-likelihood estimates.</td>
</tr>
<tr>
<td>R2</td>
<td>Coefficient of determination for each equation, obtained as the squared of the correlation coefficient between the corresponding explained variable and fitted values.</td>
</tr>
<tr>
<td>R2 pooled</td>
<td>Global coefficient of determination obtained for the set of the $G$ equations. It is computed in the same way as uniequational $R^2$ but joining the dependent variable and fitted values in single vectors instead of one vector for each equation.</td>
</tr>
<tr>
<td>Sigma</td>
<td>Estimated covariance matrix for the residuals of the $G$ equations.</td>
</tr>
<tr>
<td>fdHess</td>
<td>Logical value of fdHess argument when computing numerical covariances.</td>
</tr>
<tr>
<td>residuals</td>
<td>Residuals of the model.</td>
</tr>
<tr>
<td>df.residuals</td>
<td>Degrees of freedom for the residuals.</td>
</tr>
<tr>
<td>fitted.values</td>
<td>Estimated values for the dependent variables.</td>
</tr>
<tr>
<td>BP</td>
<td>Value of the Breusch-Pagan statistic to test the null hypothesis of diagonality among the errors of the $G$ equations.</td>
</tr>
<tr>
<td>LMM</td>
<td>Marginal Lagrange Multipliers, $LM(\rho</td>
</tr>
<tr>
<td>G</td>
<td>Number of equations.</td>
</tr>
<tr>
<td>N</td>
<td>Number of cross-sections or spatial units.</td>
</tr>
<tr>
<td>Tm</td>
<td>Number of time periods.</td>
</tr>
<tr>
<td>p</td>
<td>Number of regressors by equation (including intercepts).</td>
</tr>
<tr>
<td>Y</td>
<td>If data is NULL, vector $Y$ of the explained variables of the SUR model.</td>
</tr>
<tr>
<td>X</td>
<td>If data is NULL, matrix $X$ of the regressors of the SUR model.</td>
</tr>
<tr>
<td>W</td>
<td>Spatial weighting matrix.</td>
</tr>
<tr>
<td>zero.policy</td>
<td>Logical value of zero.policy.</td>
</tr>
<tr>
<td>interval</td>
<td>Search interval for spatial parameter.</td>
</tr>
<tr>
<td>listw_style</td>
<td>Style of neighborhood matrix $W$.</td>
</tr>
<tr>
<td>trs</td>
<td>Either NULL or vector of powered spatial weights matrix traces output by tr$W$.</td>
</tr>
<tr>
<td>insert</td>
<td>Logical value to check if is.null(trs).</td>
</tr>
</tbody>
</table>

### Control arguments

<table>
<thead>
<tr>
<th>tol</th>
<th>Numerical value for the tolerance for the estimation algorithm until convergence. Default = 1e-3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxit</td>
<td>Maximum number of iterations until convergence; it must be an integer value. Default = 200.</td>
</tr>
<tr>
<td>trace</td>
<td>A logical value to show intermediate results during the estimation process. Default = TRUE.</td>
</tr>
<tr>
<td>fdHess</td>
<td>Compute variance-covariance matrix using the numerical hessian. Suited for large samples. Default = FALSE</td>
</tr>
<tr>
<td>Imult</td>
<td>default 2; used for preparing the Cholesky decompositions for updating in the Jacobian function</td>
</tr>
<tr>
<td>super</td>
<td>if NULL (default), set to FALSE to use a simplicial decomposition for the sparse Cholesky decomposition and method &quot;Matrix&quot;, set to as.logical(NA) for method &quot;Matrix&quot;, if TRUE, use a supernodal decomposition</td>
</tr>
<tr>
<td>cheb_q</td>
<td>default 5; highest power of the approximating polynomial for the Chebyshev approximation</td>
</tr>
<tr>
<td>MC_p</td>
<td>default 16; number of random variates</td>
</tr>
<tr>
<td>MC_m</td>
<td>default 30; number of products of random variates matrix and spatial weights matrix</td>
</tr>
<tr>
<td>spamPivot</td>
<td>default &quot;MMD&quot;, alternative &quot;RCM&quot;</td>
</tr>
<tr>
<td>in_coef</td>
<td>default 0.1, coefficient value for initial Cholesky decomposition in &quot;spam_update&quot;</td>
</tr>
<tr>
<td>type</td>
<td>default &quot;MC&quot;, used with method &quot;moments&quot;; alternatives &quot;mult&quot; and &quot;moments&quot;. For use if trs is missing</td>
</tr>
<tr>
<td>correct</td>
<td>default TRUE, used with method &quot;moments&quot; to compute the Šmírnov/Anselin correction term</td>
</tr>
</tbody>
</table>
trunc  default TRUE, used with method "moments" to truncate the Smirnov/Anselin correction term
SE_method  default "LU", may be "MC"
nrho  default 200, as in SE toolbox; the size of the first stage lndet grid; it may be reduced to for example 40
interp  default 2000, as in SE toolbox; the size of the second stage lndet grid
SElndet  default NULL, may be used to pass a pre-computed SE toolbox style matrix of coefficients and their lndet values to the methods
LU_order  default FALSE; used in "LU_prepermutate", note warnings given for lu method
pre_eig  default NULL; may be used to pass a pre-computed vector of eigenvalues

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Jesus Mur  <jmur@unizar.es>

References


See Also

spsur3sls, lagsarl, lmtestspur, wald_betas, lr_betas

Examples

############################################################################
####### CROSS SECTION DATA (G>1; Tm=1) ########
############################################################################

rm(list = ls()) # Clean memory
data(spc)
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
spcsur.sim <- spsurml(formula = Tformula, data = spc, type = "sim")
summary(spcsur.sim)
# All the coefficients in a single table.
print(spcsur.sim)
# Plot of the coefficients of each equation in different graphs
plot(spcsur.sim)

## A SUR-SLX model
## (listw argument can be either a matrix or a listw object )
spcsur.slx <- spsurml(formula = Tformula, data = spc, type = "slx",
    listw = Wspc, Durbin = TRUE)
summary(spcsur.slx)
# All the coefficients in a single table.
print(spcsur.slx)
# Plot of the coefficients in a single graph
if (require(gridExtra)) {
    pl <- plot(spcsur.slx, viewplot = FALSE)
    grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
        nrow = 2)
}

## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_spsurml, package="spsur")

#### A SUR-SLM model
spcsur.slm <- spsurml(formula = Tformula, data = spc, type = "slm",
    listw = Wspc)
summary(spcsur.slm)
if (require(gridExtra)) {
    pl <- plot(spcsur.slm, viewplot = FALSE)
    grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
        pl$pldeltas, nrow = 3)
}

#### A SUR-SDM model
spcsur.sdm <- spsurml(formula = Tformula, data = spc, type = "sdm",
```r
listw = Wspc
summary(spcsur.sdm)
print(spcsur.sdm)
if (require(gridExtra)) {
  pl <- plot(spcsur.sdm, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
  pl$pldeltas, nrow = 3)
}

## A SUR-SDM model with different spatial lags in each equation
TformulaD <- ~ UN83 + NMR83 + SMSA | UN80
spcsur.sdm2 <- spsurml(formula = Tformula, data = spc, type = "sdm",
  listw = Wspc, Durbin = TformulaD)
summary(spcsur.sdm2)
if (require(gridExtra)) {
  pl <- plot(spcsur.sdm2, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
  pl$pldeltas, nrow = 3)
}

########################################################################
#### CLASSIC PANEL DATA G=1; Tm>1 ####
########################################################################

### Example 2: Homicides + Socio-Economics (1960–90)
### Homicides and selected socio-economic characteristics for continental
### U.S. counties.
### \url{https://geodacenter.github.io/data-and-lab/ncovr/}

### It usually requires 1-2 minutes maximum...
rm(list = ls()) # Clean memory
### Read NCOVR.sf object
data(NCOVR, package = "spsur")
nbncovr <- spdep::poly2nb(NCOVR.sf, queen = TRUE)
### Some regions with no links...
lwncovr <- spdep::nb2listw(nbncovr, style = "W", zero.policy = TRUE)
Tformula <- HR80 | HR90 ~ PS80 + UE80 | PS90 + UE90
### A SUR-SIM model
NCOVRSUR.sim <- spsurml(formula = Tformula, data = NCOVR.sf, type = "sim")
summary(NCOVRSUR.sim)
if (require(gridExtra)) {
  pl <- plot(NCOVRSUR.sim, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]], nrow = 3)
}
### A SUR-SLX model
NCOVRSUR.slx <- spsurml(formula = Tformula, data = NCOVR.sf, type = "slx",
  listw = lwncovr, zero.policy = TRUE)
print(NCOVRSUR.slx)
if (require(gridExtra)) {
  pl <- plot(NCOVRSUR.slx, viewplot = FALSE)
}
```
grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]], nrow = 2)
}

### A SUR-SLM model
### method = "Matrix" (Cholesky) instead of "eigen"
### (fdHess = TRUE to compute numerical covariances)
NCOVRSUR.slm <- spsurml(formula = Tformula, data = NCOVR.sf,
type = "slm", listw = lwncovr, method = "Matrix",
zero.policy = TRUE, control = list(fdHess = TRUE))
summary(NCOVRSUR.slm)

if (require(gridExtra)) {
  pl <- plot(NCOVRSUR.slm, viewplot = FALSE)
  grid.arrange(pl$lplbetas[[1]], pl$lplbetas[[2]],
               pl$pldeltas, nrow = 3)
}
# LR test for nested models
anova(NCOVRSUR.sim, NCOVRSUR.slm)

spsurtime

Estimation of SUR models for simple spatial panels (G=1).

Description

This function estimates SUR models for simple spatial panel datasets. `spsurtime` is restricted, specifically, to cases where there is only one equation, \( G=1 \), and a varying number of spatial units, \( N \), and time periods, \( Tm \). The SUR structure appears in form of serial dependence among the error terms corresponding to the same spatial unit. Note that it is assumed that all spatial units share a common pattern of serial dependence.

The user can choose between different types of spatial specifications, as described below, and the estimation algorithms allow for the introduction of linear restrictions on the \( \beta \) parameters associated to the regressors. The spatial panels with SUR structure can be estimated by maximum-likelihood methods or three-stages least squares procedures, using spatial instrumental variables.

Usage

```
spsurtime(formula, data, time, na.action,
          listw = NULL, type = "slm", Durbin = NULL,
          method = "eigen", fit_method = "ml", maxlagW = NULL,
          zero.policy = NULL, interval = NULL, trs = NULL,
          R = NULL, b = NULL, demean = FALSE, control = list() )
```

Arguments

- `formula`: An object type `Formula` similar to objects created with the package `Formula` describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.

Description

This function estimates SUR models for simple spatial panel datasets. `spsurtime` is restricted, specifically, to cases where there is only one equation, \( G=1 \), and a varying number of spatial units, \( N \), and time periods, \( Tm \). The SUR structure appears in form of serial dependence among the error terms corresponding to the same spatial unit. Note that it is assumed that all spatial units share a common pattern of serial dependence.

The user can choose between different types of spatial specifications, as described below, and the estimation algorithms allow for the introduction of linear restrictions on the \( \beta \) parameters associated to the regressors. The spatial panels with SUR structure can be estimated by maximum-likelihood methods or three-stages least squares procedures, using spatial instrumental variables.

Usage

```
spsurtime(formula, data, time, na.action,
          listw = NULL, type = "slm", Durbin = NULL,
          method = "eigen", fit_method = "ml", maxlagW = NULL,
          zero.policy = NULL, interval = NULL, trs = NULL,
          R = NULL, b = NULL, demean = FALSE, control = list() )
```

Arguments

- `formula`: An object type `Formula` similar to objects created with the package `Formula` describing the equations to be estimated in the model. This model may contain several responses (explained variables) and a varying number of regressors in each equation.
data  An object of class data.frame or a matrix.
time  Time variable.
n.a. action  A function (default options("na.action")), can also be na.omit or na.exclude with consequences for residuals and fitted values. It may be necessary to set zero.policy to TRUE because this subsetting may create no-neighbour observations.
listw  A listw object created for example by nb2listw from spatialreg package; if nb2listw not given, set to the same spatial weights as the listw argument. It can also be a spatial weighting matrix of order \((N\times N)\) instead of a listw object. Default = NULL.
type  Type of spatial model specification: "sim", "slx", "slm", "sem", "sdm", "sdem", "sarar" or "gnm". Default = "sim".
Durbin  If a formula object and model is type "sdm", "sdem" or "slx" the subset of explanatory variables to lag for each equation.
method  Similar to the corresponding parameter of lagsarlm function in spatialreg package. "eigen" (default) - the Jacobian is computed as the product of \((1 - \rho \times \text{eigenvalue})\) using eigenw, and "spam" or "Matrix_J" for strictly symmetric weights lists of styles "B" and "C", or made symmetric by similarity (Ord, 1975, Appendix C) if possible for styles "W" and "S", using code from the spam or Matrix packages to calculate the determinant; "Matrix" and "spam_update" provide updating Cholesky decomposition methods; "LU" provides an alternative sparse matrix decomposition approach. In addition, there are "Chebyshev" and Monte Carlo "MC" approximate log-determinant methods; the Smirnov/Anselin (2009) trace approximation is available as "moments". Three methods: "SE_classic", "SE_whichMin", and "SE_interp" are provided experimentally, the first to attempt to emulate the behaviour of Spatial Econometrics toolbox ML fitting functions. All use grids of log determinant values, and the latter two attempt to ameliorate some features of "SE_classic".
fit_method  Method of estimation for the spatial panel SUR model, either ml or 3sls. Default = ml.
maxlagW  Maximum spatial lag order of the regressors employed to produce spatial instruments for the spatial lags of the explained variables. Default = 2. Note that in case of type = "sdm", the default value for maxlagW is set to 3 because the first lag of the regressors, \(WX_{tg}\), can not be used as spatial instruments.
zero.policy  Similar to the corresponding parameter of lagsarlm function in spatialreg package. If TRUE assign zero to the lagged value of zones without neighbours, if FALSE assign NA - causing spsurml() to terminate with an error. Default = NULL.
interval  Search interval for autoregressive parameter. Default = NULL.
trs  Either NULL or vector of powered spatial weights matrix traces output by trW. Default = NULL.
R  A row vector of order \(lxpr\) with the set of \(r\) linear constraints on the beta parameters. The first restriction appears in the first \(p\) terms, the second restriction in the next \(p\) terms and so on. Default = NULL.
**b**  
A column vector of order \((rx1)\) with the values of the linear restrictions on the *beta* parameters. Default = NULL.

**demean**  
Logical value to allow for the demeaning of panel data. In this case, `spsurml` subtracts the individual mean to each spatial or cross-sectional unit. Default = FALSE.

**control**  
list of additional arguments.

**Details**

Function `spsurtime` only admits a formula, created with `Formula` and a dataset of class `data.frame` or `matrix`. That is, the data cannot be uploaded using data matrices \(Y\) and \(X\) provided for other functions in this package.

The argument `time` selects the variable, in the `data.frame`, associated to the time dimension in the panel dataset. Then `spsurtime` operates as in Anselin (1988), that is, each cross-section is treated as if it were an equation in a SUR model, which now has \(Tm\) ’equations’ and \(N\) individuals.

The SUR structure appears because there is serial dependence in the errors of each individual in the panel. The serial dependence in the errors is not parameterized, but estimated non-parametrically in the \(Sigma\) covariance matrix returned by the function. An important constraint to mention is that the serial dependence assumed to be the same for all individuals in the sample. Serial dependence among individuals is excluded from Anselin approach.

**Value**

An `spsur` object with the output of the maximum-likelihood or three-stages least-squares estimation of the spatial panel SUR model.

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Jesus Mur  
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**References**


See Also

spsurml, spsur3sls, wald_betas, wald_deltas, lmtestspsur, lr_betas

Examples

####################################################
########## PANEL DATA (G=1; Tm>1) ######
####################################################

### Example 1:
rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
N <- nrow(spc)
Tm <- 2
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
WAGE <- c(spc$WAGE83, spc$WAGE81)
UN <- c(spc$UN83, spc$UN80)
NMR <- c(spc$NMR83, spc$NMR80)
SMSA <- c(spc$SMSA, spc$SMSA)
pspc <- data.frame(index_indiv, index_time, WAGE, UN, NMR, SMSA)
form_pspc <- WAGE ~ UN + NMR + SMSA
form2_pspc <- WAGE | NMR ~ UN | UN + SMSA

# SLM
pspc_slm <- spsurtime(formula = form_pspc, data = pspc,
                      listw = lwspc,
                      time = pspc$index_time,
                      type = "slm", fit_method = "ml")
summary(pspc_slm)

pspc_slm2 <- spsurtime(formula = form2_pspc, data = pspc,
                       listw = lwspc,
                       time = pspc$index_time,
                       type = "slm", fit_method = "ml")
summary(pspc_slm2)

### VIP: The output of the whole set of the examples can be examined
### by executing demo(demo_spsurtime, package="spsur")

### Example 2:
rm(list = ls()) # Clean memory
## Read NCOVR.sf object
data(NCOVR, package="spsur")
```r
nbncovr <- spdep::poly2nb(NCOVR.sf, queen = TRUE)
### Some regions with no links...
lwncovr <- spdep::nb2listw(nbncovr, style = "W", zero.policy = TRUE)
N <- nrow(NCOVR.sf)
Tm <- 4
index_time <- rep(1:Tm, each = N)
index_indiv <- rep(1:N, Tm)
pHR <- c(NCOVR.sf$HR60, NCOVR.sf$HR70, NCOVR.sf$HR80, NCOVR.sf$HR90)
pPS <- c(NCOVR.sf$PS60, NCOVR.sf$PS70, NCOVR.sf$PS80, NCOVR.sf$PS90)
pUE <- c(NCOVR.sf$UE60, NCOVR.sf$UE70, NCOVR.sf$UE80, NCOVR.sf$UE90)
pNCOVR <- data.frame(indiv = index_indiv, time = index_time,
HR = pHR, PS = pPS, UE = pUE)

form_pHR <- HR ~ PS + UE

## SIM
pHR_sim <- spsurtime(formula = form_pHR, data = pNCOVR,
time = pNCOVR$time, type = "sim", fit_method = "ml")
summary(pHR_sim)
## SLM by 3SLS.
pHR_slm <- spsurtime(formula = form_pHR, data = pNCOVR,
listw = lwncovr,
time = pNCOVR$time, type = "slm",
fit_method = "3sls")
summary(pHR_slm)

############################# Wald tests about betas in spatio-temporal models
### H0: equal betas for PS in equations 1, 3 and 4.
R <- matrix(0, nrow = 2, ncol = 12)
## nrow = number of restrictions
## ncol = number of beta parameters
R[1, 2] <- 1; R[1, 8] <- -1 # PS beta coefficient in equations 1 equal to 3
R[2, 2] <- 1; R[2, 11] <- -1 # PS beta coefficient in equations 1 equal to 4
b <- matrix(0, nrow=2, ncol=1)
wald_betas(pHR_sim , R = R , b = b) # SIM model
wald_betas(pHR_slm , R = R , b = b) # SLM model

############################# Wald tests about spatial-parameters in
############################# spatio-temporal models
### H0: equal rhos in slm model for equations 1 and 2.
R2 <- matrix(0, nrow = 1, ncol = 4)
R2[1, 1] <- -1; R2[1, 2] <- -1
b2 <- matrix(0, nrow = 1, ncol = 1)
wald_deltas(pHR_slm, R = R2, b = b2)
```

---

**summary.spsur**

Summary of estimated objects of class spsur.
Description

This function summarizes estimated spsur objects. The tables in the output include basic information for each equation. The report also shows other complementary results corresponding to the SUR model like the $(G \times G)$ covariance matrix of the residuals of the equations of the SUR, the estimated log-likelihood, the Breusch-Pagan diagonality test or the Marginal Lagrange Multiplier, LMM, tests of spatial dependence.

Usage

```r
## S3 method for class 'spsur'
summary(object, ...)
```

Arguments

- **object**: An spsur object estimated using spsurml, spsur3sls or spsurtime functions.

- **...**: further arguments passed to or from other methods.

Value

An object of class summary.spsur

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See Also

print.summary.spsur; spsurml; spsur3sls.

Examples

# See examples for \code{\link{spsurml}} or  
# \code{\link{spsur3sls}} functions.
Description

The function `wald_betas` can be seen as a complement to the restricted estimation procedures included in the functions `spsurml` and `spsur3sls`. `wald_betas` obtains Wald tests for sets of linear restrictions on the coefficients $\beta$ of the SUR model. The restrictions may involve coefficients of the same equation or coefficients from different equations. The function has great flexibility in this respect. Note that `wald_betas` is more general than `lr_betas` in the sense that the last function only allows to test for restrictions of homogeneity of subsets of $\beta$ coefficients among the different equations in the SUR model, and in a maximum-likelihood framework.

In order to work with `wald_betas`, the model on which the linear restrictions are to be tested needs to exists as an `spsur` object. Using the information contained in the object, `wald_betas` obtains the corresponding Wald estatistic for the null hypotheses specified by the user through the $R$ row vector and $b$ column vector, used also in `spsurml` and `spsur3sls`. The function shows the value of the Wald test statistics and its associated p-values.

Usage

```r
wald_betas (obj , R , b)
```

Arguments

- **obj**: An `spsur` object created by `spsurml`, `spsur3sls` or `spsurtime`.
- **R**: A row vector of order $(1 \times Pr)$ showing the set of $r$ linear constraints on the $\beta$ parameters. The first restriction appears in the first $K$ terms in $R$, the second restriction in the next $K$ terms and so on.
- **b**: A column vector of order $(r \times 1)$ with the values of the linear restrictions on the $\beta$ parameters.

Value

Object of `htest` class including the Wald statistic, the corresponding p-value, the degrees of freedom and the values of the sample estimates.

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References


See Also

`spsurml`, `spsur3sls`, `lr_betas`

Examples

```r
## VIP: The output of the whole set of the examples can be examined
## by executing demo(demo_wald_betas, package="spsur")

###################################
##### CROSS SECTION DATA (G=1; Tm>1) ######
###################################

### Example 1: Spatial Phillips-Curve. Anselin (1988, p. 203)
rm(list = ls()) # Clean memory
data(spc)
lwspc <- spdep::mat2listw(Wspc, style = "W")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
### Estimate SUR-SLM model
spcsur.slm <- spsurml(formula = Tformula, data = spc,
type = "slm", listw = lwspc)
summary(spcsur.slm)
### H_0: equality between SMSA cоefficients in both equations.
R1 <- matrix(c(0,0,0,1,0,0,0,-1), nrow=1)
b1 <- matrix(0, ncol=1)
wald_betas(spcsur.slm, R = R1, b = b1)
```

Function **wald_deltas** obtains Wald tests for linear restrictions on the spatial coefficients of a SUR model that has been estimated previously through the function `spsurml`. The restrictions can affect to coefficients of the same equation (i.e., $\lambda_g = \rho_g$ for all $g$) or can involve coefficients from different equations (i.e., $\lambda_g = \lambda_h$). The function has great flexibility in this respect. Note that **wald_deltas** only works in a maximum-likelihood framework.
In order to work with `wald_betas`, the model on which the linear restrictions are to be tested needs to exist as a `spsur` object. Using the information contained in the object, `wald_deltas` obtains the corresponding Wald statistic for the null hypotheses specified by the user through the `R` row vector and `b` column vector discussed, used also in `spsurml`. The function shows the resulting Wald test statistics and their corresponding p-values.

**Usage**

```r
wald_deltas (obj, R, b)
```

**Arguments**

- `obj`: An `spsur` object created by `spsurml`, `spsur3sls` or `spsurtime`.
- `R`: A row vector of order `(1xGr)` or `(1x2Gr)` showing the set of `r` linear constraints on the spatial parameters. The last case is reserved to "sarar" models where there appear `G` parameters `λ_g` and `G` parameters `ρ_g`, `2G` spatial in total. The first restriction appears in the first `G` terms in `R` (`2G` for the "sarar" case), the second restriction in the next `G` terms (`2G` for the "sarar" case) and so on.
- `b`: A column vector of order `(rx1)` with the values of the linear restrictions on the `β` parameters.

**Value**

Object of `htest` including the Wald statistic, the corresponding p-value, the degrees of freedom and the values of the sample estimates.

**Author(s)**

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**See Also**

`spsurml, spsur3sls`

**Examples**

```r
#################################################
######## CROSS SECTION DATA (G>1; Tm=1) ########
#################################################
rm(list = ls()) # Clean memory
data(spc, package = "spsur")
lwspc <- spdep::mat2listw(Wspc, style = "W")
Tformula <- WAGE83 | WAGE81 ~ UN83 + NMR83 + SMSA | UN80 + NMR80 + SMSA
```
## Estimate SUR-SLM model

```r
spcsur.slm <- spsurml(formula = Tformula, data = spc,
                      type = "slm", listw = lwspc)
summary(spcsur.slm)
```

## H₀: equality of the lambda parameters of both equations.

```r
R1 <- matrix(c(1,-1), nrow=1)
b1 <- matrix(0, ncol=1)
wald_deltas(spcsur.slm, R = R1, b = b1)
```

---

**Wspc**

*Spatial weight matrix for South-West Ohio Counties to estimate Spatial Phillips-Curve*

### Description

A spatial weight matrix row-standardized based on first order contiguity criterion.

### Usage

- `Wspc`

### Format

A row-standardized squared matrix with 25 rows and columns. The rows and columns follow the same order than provinces included in `spc` data frame.

### Source

Anselin (1988, p. 207)

### References

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