# Package ‘ssfa’

June 10, 2015

**Type** Package  
**Title** Spatial Stochastic Frontier Analysis  
**Version** 1.1  
**Date** 2015-06-09  
**Author** Elisa Fusco, Francesco Vidoli  
**Maintainer** Elisa Fusco <fusco_elisa@libero.it>

**Description** Spatial Stochastic Frontier Analysis (SSFA) is an original method for controlling the spatial heterogeneity in Stochastic Frontier Analysis (SFA) models, for cross-sectional data, by splitting the inefficiency term into three terms: the first one related to spatial peculiarities of the territory in which each single unit operates, the second one related to the specific production features and the third one representing the error term.

**Depends** Matrix, maxLik, spdep, sp  
**License** GPL-3  
**Suggests** R.rsp  
**VignetteBuilder** R.rsp  
**NeedsCompilation** no  
**Repository** CRAN

**Date/Publication** 2015-06-10 01:02:51

## R topics documented:

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ssfa-package</td>
<td>2</td>
</tr>
<tr>
<td>eff.ssfa</td>
<td>2</td>
</tr>
<tr>
<td>fitted.ssfa</td>
<td>3</td>
</tr>
<tr>
<td>Italian_W</td>
<td>4</td>
</tr>
<tr>
<td>L_hNV</td>
<td>5</td>
</tr>
<tr>
<td>L_hNV_rho</td>
<td>5</td>
</tr>
<tr>
<td>plot_fitted</td>
<td>6</td>
</tr>
<tr>
<td>plot_moran</td>
<td>7</td>
</tr>
<tr>
<td>residuals.ssfa</td>
<td>8</td>
</tr>
<tr>
<td>ssfa</td>
<td>9</td>
</tr>
</tbody>
</table>
Description

The package implements the Spatial Stochastic Frontier model for cross-sectional data introduced by Fusco and Vidoli (2013). The method controls spatial heterogeneity in SFA models by splitting the inefficiency term into three parts: the first one related to spatial peculiarities of the territory in which each single unit operates, the second one related to the specific production features and the third one representing the error term.

Details

Package: ssfa
Type: Package
Version: 1.1
Date: 2015-06-09
License: GPL-3

Author(s)

Elisa Fusco, Francesco Vidoli
Maintainer: Elisa Fusco <fusco_elisa@libero.it>

References


Description

This function returns the technical efficiency of each producer (without local spatial effects) calculated by the Battese and Coelli (1988) formulation modified by using an autoregressive specification in the inefficiency term \( u \).
Usage

eff.ssfa(object, ...)

Arguments

object an object of class ssfa.

... further arguments for methods.

Value

Technical efficiency of each producer (without local spatial effects).

References


See Also

u.ssfa

Examples

library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
form = "production", par_rho=TRUE)
eff <- eff.ssfa(ssfa)

fitted.ssfa SSFA fitted values

Description

This function returns the fitted values of the original data used to estimate the SSFA model.

Usage

## S3 method for class 'ssfa'
fitted(object, ...)

Arguments

object an object of class ssfa.

... further arguments for methods.

Examples

library(ssfa)
data(SSFA_example_data)
data(italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
             form = "production", par_rho=TRUE)
fitted.ssfa(ssfa)

---

Italian_W Italian provinces spatial weights matrix example

Description

This is an example dataset that contains the 107 Italian provinces contiguity matrix (year 2008).

Usage

data(italian_W)

Format

A data frame with 107 x 107 row-standardized distances between observations (Italian provinces).

References

http://www.istat.it/it/archivio/104317#confini.

Examples

data(italian_W)
L_hNV

SFA half-normal log likelihood function

Description

This function is used to estimate the parameters of the classical SFA model where half-normal distribution of inefficiency term is assumed.

Usage

L_hNV(p, y = y, X = X, sc = sc)

Arguments

- p: a vector with the parameters to be estimated.
- y: the dependent variable.
- X: the model matrix.
- sc: specifies the form of the frontier model (-1 = cost, 1 = production).

Value

Value of the SFA log likelihood function.

L_hNV_rho

SSFA half-normal log likelihood function

Description

This function is used to estimate the parameters of the SSFA model where half-normal distribution of inefficiency term is assumed.

Usage

L_hNV_rho(p, y = y, X = X, sc = sc, w = w, sigm2sar = sigm2sar)

Arguments

- p: a vector with the parameters to be estimated.
- y: the dependent variable.
- X: the model matrix.
- sc: specifies the form of the frontier model (-1 = cost, 1 = production).
- w: the spatial weight matrix.
- sigm2sar: is the variance of the spatial correlated part of the inefficiency term estimated into ssfa.fit function.
Value

Value of the SSFA log likelihood function.

Note

Please note that `sigma2_sar` is not a free parameter because it is estimated into the `ssfa.fit` function.

See Also

`ssfa`

---

### Description

This function allows to plot the data and the fitted values obtained by SSFA model.

### Usage

```r
plot_fitted(x, y, object, xlab, ylab, main, ...)
```

### Arguments

- `x` 
the x coordinates of points in the plot.
- `y` 
the y coordinates of points in the plot.
- `object` 
an object of class `ssfa`.
- `xlab` 
a title for the x axis.
- `ylab` 
a title for the y axis.
- `main` 
an overall title for the plot.
- `...` 
arguments to be passed to methods, such as graphical parameters (see `par`).

### See Also

`plot`
Examples

```r
library(ssfa)
data(SSFA_example_data)
data(italian_w)

#### SFA and SSFA comparison
sfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=italian_W,
            form = "production", par_rho=FALSE)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=italian_W,
            form = "production", par_rho=TRUE)
sfa_fitted <- fitted.ssfa(sfa)
plot_fitted(SSFA_example_data$log_x, SSFA_example_data$log_y, ssfa)
lines(sort(SSFA_example_data$log_x), sfa_fitted[order(SSFA_example_data$log_x)], col="red")
```

Description

This function allows to plot the residuals of the object against their spatially lagged values, augmented by reporting the summary of influence measures for the linear relationship between the data and the lag.

Usage

```r
plot_moran(x, main, xlab, ylab, labels, listw, ...)
```

Arguments

- **x** an object of class ssfa.
- **main** an overall title for the plot.
- **xlab** a label for the x axis.
- **ylab** a label for the y axis.
- **labels** character labels for points with high influence measures, if set to FALSE, no labels are plotted for points with large influence.
- **listw** a listw object from nb2listw (see `nb2listw`).
- **...** arguments to be passed to methods, such as graphical parameters (see `par`).

References

residuals.ssfa

See Also

moran.plot

Examples

library(ssfa)
data(SSFA_example_data)
data(italian_W)

### SFA and SSFA comparison ###
sfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
form = "production", par_rho=FALSE)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data, data_w=Italian_W,
form = "production", par_rho=TRUE)
moran.test(residuals.ssfa(sfa), sfa$list_w)
moran.test(residuals.ssfa(ssfa), ssfa$list_w)

plot_moran(sfa, listw=sfa$list_w)
plot_moran(ssfa, listw=ssfa$list_w)

residuals.ssfa SSFA residuals

Description

This function returns the residuals of the fitted SSFA model.

Usage

## S3 method for class 'ssfa'
residuals(object, ...)

Arguments

object an object of class ssfa.
...

further arguments for methods.

Examples

library(ssfa)
data(SSFA_example_data)
data(italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
data_w=Italian_W, form = "production", par_rho=TRUE)
residuals.ssfa(ssfa)
**SSFA**

*Spatial stochastic frontier estimation*

**Description**

This function estimates the Spatial Stochastic Frontier model introduced by Fusco and Vidoli (2013) in the following form:

\[
\log(y_i) = \log(f(x_i; \beta_i)) + v_i - u_i
\]

\[
u_i = \rho \sum_i w_i u_i + \tilde{u}_i
\]

where \( y_i \) are the outputs, \( x_i \) the inputs, \( v_i \) the stochastic noise, \( u_i \) the inefficiency term, \( \rho \) the spatial lag, \( w_i \) a standardized row of the spatial weights matrix and \( \tilde{u}_i \) the stochastic noise of the inefficiency term.

**Usage**

ssfa(formula, data = NULL, data_w = NULL, intercept = TRUE, pars = NULL, par_rho = TRUE, form = "cost")

**Arguments**

- **formula**: an object of class formula (or one that can be coerced to that class): a symbolic description of the model to be fitted.
- **data**: an optional data frame containing the variables in the model.
- **data_w**: a data frame containing the spatial weight matrix.
- **intercept**: logical. If true the model includes intercept.
- **pars**: initial values for the parameters to be estimated.
- **par_rho**: logical. If true the function estimates the Spatial Stochastic Frontier (SSFA) otherwise the classical Stochastic Frontier (SFA).
- **form**: specifies the form of the frontier model as "cost" or "production".

**Value**

ssfa returns the following objects of class ssfa:

- **y**: the dependent variable.
- **x**: the covariates.
- **X**: the model matrix.
- **coef**: the estimated coefficients.
- **sc**: the form of the frontier model estimated (-1 = cost, 1 = production).
- **hess**: a symmetric matrix giving an estimate of the Hessian at the solution found.
- **logLik**: the value of the log likelihood function.
ols

the linear model for the LR-test.

sigmau2

the estimation of sigmau2 (only if par_rho=FALSE): value of inefficiency variance.

sigma2_dmumu

the estimation of sigmau2_dmumu (only if par_rho=TRUE): value of the part of
the inefficiency variance due to DMU’s specificities.

sigma2_sar

the estimation of sigma2_sar: value of the part of the inefficiency variance due
to the spatial correlation.

sigmav2

the estimation of sigmav2: value of the stochastic error variance.

sigma2

the estimation of sigma2: value of the total variance.

rho

the estimation of the spatial lag parameter rho.

fun

the distribution of the inefficiency term u.

list_w

a listw object from nb2listw (See nb2listw).

Note

NOTE 1: In this version the distribution of the inefficiency term u is only “half-normal”.

NOTE 2: The method used to maximize the log likelihood function is the Newton-Raphson. Please
see the R function maxNR of the maxLik package for details (Henningsen and Toomet (2011)).

NOTE 3: Please note that the classical SFA inefficiency variance sigma2, in the SSFA, is decom-
posed into sigma2_dmumu and sigma2_sar, respectively the part of inefficiency variance due to
DMU’s specificities and to the spatial dependence, i.e. sigma2 = sigma2_dmumu + sigma2_sar
and consequently the total variance is given by sigma2 = sigma2_dmumu + sigma2_sar + sigmav2.

Author(s)

Fusco E. and Vidoli F.

References


Fusco, E. and Vidoli, F. (2013). Spatial stochastic frontier models: controlling spatial global and

Press.


Examples

library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
### SSFA total variance decomposition

```r
## SSFA example_data
sigma2 = ssfa$sigma2_dmu + ssfa$sigma2_sar + ssfa$sigma2
```

---

**Description**

The dataset contains the simulated data used by Fusco and Vidoli (2013) to test the model. Data Generating Process (DGP) follows the construction criteria proposed by Banker and Natarajan (2008), also used by Johnson and Kuosmanen (2011), with the addition of a strong spatial correlation in the inefficiency term through a spatial lag parameter and a contiguity matrix (107 Italian provinces contiguity matrix, year 2008).

**Usage**

```r
data(SSFA_example_data)
```

**Format**

A data frame with 107 observations (Italian provinces) and 2 variables:

- `dmu` the Decision Making Unit name.
- `log_x` the input vector (already in logarithmic form).
- `log_y` the output vector (already in logarithmic form).

**References**


**Examples**

```r
data(SSFA_example_data)
```
**Description**

The function `print.ssfa` is used to display the values of SFA and SSFA estimated coefficients. In particular:
- for SFA the function displays the intercept, the regressors beta coefficients, the inefficiency variance $\sigma_u^2$, the stochastic error variance $\sigma_v^2$ and the total variance $\sigma^2$;
- for SSFA the function displays, in addition, the decomposition of the inefficiency variance into $\sigma_u^2_{dmu}$ and $\sigma_u^2_{sar}$, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, and finally, the spatial lag parameter $\rho$.

The function `summary.ssfa` is used to display the summary results of SFA and SSFA. In particular:
- for SFA the summary shows the estimation of SFA coefficients (intercept, beta coefficients, $\sigma_u^2$ and $\sigma_v^2$) and others useful information as the total variance $\sigma^2$, the inefficiency parameter $\lambda$ ($\sigma_u/\sigma_v$), the Moran $I$ statistic, the mean of efficiency, the LR-test and the AIC values;
- for SSFA the summary shows, in addition, the decomposition of the inefficiency variance into $\sigma_u^2_{dmu}$ and $\sigma_u^2_{sar}$ and the spatial lag parameter $\rho$.

**Usage**

```r
## S3 method for class 'ssfa'
print(x, ...)  
## S3 method for class 'ssfa'
summary(object, ...)
```

**Arguments**

- `x` an object of class `ssfa`.
- `object` an object of class `ssfa`.
- `...` further arguments for methods.

**Note**

Please note that the classical SFA inefficiency variance $\sigma_u^2$, in the SSFA, is decomposed into $\sigma_u^2_{dmu}$ and $\sigma_u^2_{sar}$, respectively the part of inefficiency variance due to DMU's specificities and to the spatial dependence, i.e. $\sigma_u^2 = \sigma_u^2_{dmu} + \sigma_u^2_{sar}$ and consequently the total variance is given by $\sigma^2 = \sigma_u^2_{dmu} + \sigma_u^2_{sar} + \sigma_v^2$.

**References**


Examples

```r
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
            data_w=Italian_W, form = "production", par_rho=TRUE)

print(ssfa)
summary(ssfa)
```

<table>
<thead>
<tr>
<th>u.ssfa</th>
<th>SSFA inefficiency</th>
</tr>
</thead>
</table>

Description

This function returns the specific inefficiency of each producer (without local spatial effects) calculated by the Jondrow et al. (JLMS) (1982) formulation modified by using an autoregressive specification in the inefficiency term.

Usage

```
u.ssfa(object, ...)
```

Arguments

- `object` an object of class `ssfa`.
- `...` further arguments for methods.

Value

Inefficiency of each producer (without local spatial effects).

References


See Also

`eff.ssfa`
Examples

```r
library(ssfa)
data(SSFA_example_data)
data(Italian_W)
ssfa <- ssfa(log_y ~ log_x, data = SSFA_example_data,
             data_w=Italian_W, form = "production", par_rho=TRUE)
ineff <- u.ssfa(ssfa)
```
Index

*Topic **datasets**
  Italian_W, 4
  SSFA_example_data, 11

*Topic **package**
  ssfa-package, 2

eff.ssfa, 2, 13
fitted.ssfa, 3
Italian_W, 4
L_hNV, 5
L_hNV_rho, 5
maxNR, 10
moran.plot, 8

nb2listw, 7, 10
par, 6, 7
plot, 6
plot_fitted, 6
plot_moran, 7
print.ssfa(summary), 12
print.summary.ssfa(summary), 12

residuals.ssfa, 8

ssfa, 6, 9
ssfa-package, 2
SSFA_example_data, 11
summary, 12
summary.ssfa(summary), 12

u.ssfa, 3, 13