The goal of the `estima` function is to estimate the coefficients of the two centered autologistic regression:

$$\text{logit}(p_{i,t}) = \chi_{i,t}^{T} \beta + \beta_{\text{past}} \sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1} + \rho_1 \sum_{j \in N_{i}} Z_{j,t-1}^{*} + \rho_2 Z_{i,t-1}$$

$$\Leftrightarrow p_{i,t} = \frac{\exp(\chi_{i,t}^{T} \beta + \beta_{\text{past}} \sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1} + \rho_1 \sum_{j \in N_{i}} Z_{j,t-1}^{*} + \rho_2 Z_{i,t-1})}{1 + \exp(\chi_{i,t}^{T} \beta + \beta_{\text{past}} \sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1} + \rho_1 \sum_{j \in N_{i}} Z_{j,t-1}^{*} + \rho_2 Z_{i,t-1})}$$

where $Z_{i,t}$ is a binary variable of parameter $p_{i,t}$, $N_{i}$ is the neighborhood of the site $i$ for the instantaneous spatial dependence, $N_{i}^{\text{past}}$ is the neighborhood of the site $i$ for the spatio-temporal dependence (spread of the illness) and $Z_{i,t-1}^{*}$ is given by:

$$Z_{i,t}^{*} = Z_{i,t} - \frac{\exp(\chi_{i,t}^{T} \beta + \beta_{\text{past}} \sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1} + \rho_2 Z_{i,t-1})}{1 + \exp(\chi_{i,t}^{T} \beta + \beta_{\text{past}} \sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1} + \rho_2 Z_{i,t-1})}.$$ 

Estimation uses the pseudo-likelihood:

$$L(\beta, \beta_{\text{past}}, \rho_1, \rho_2) = \prod_{t=1}^{T} \prod_{1 \leq i \leq n} (p_{i,t})^{z_{i,t}}(1 - p_{i,t})^{1-z_{i,t}}.$$ 

For more detail see Gegout-Petit, Guérin-Dubrana, Li, 2019.

The parameters of spatio-temporal dependence $\rho_1, \rho_2, \beta_{\text{past}}$ can be interpreted as practical biological processes:

— Instantaneous spatial dependence $\rho_1$. It quantifies the spatial autocorrelation between neighbours for the occurrence of the event at each time $t$.

— Temporal dependence $\rho_2$. It quantifies the temporal dependence on the previous year’s status.

— Coefficient $\beta_{\text{past}}$: it quantifies the spread of the illness coming from the previous year’s status of the neighbours.

The function `estima` estimates the parameters with different possibilities for $\beta_{\text{past}}$ and $\sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1}$:

if "covpast = FALSE" : estimates the parameter $\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$ and $\chi_{i,t}^{T} = \begin{pmatrix} 1 \\ x_{1,i,t}^{1} \\ x_{1,i,t}^{2} \\ x_{1,i,t}^{3} \end{pmatrix}$ where $x_{j,i,t}^{j} \forall j \in \{1, 2, 3\}$ is a spatio-temporal covariate. There can be 0, 1, 2 or 3 covariates. In this case, there is no regression on $\sum_{j \in N_{i}^{\text{past}}} Z_{j,t-1}$ ($\beta_{\text{past}} = 0$).

if "covpast = TRUE" : the function estimates the parameters $\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$ and $\beta_{\text{past}}$. 