Package ‘trend’

January 13, 2020

Version 1.1.2
Date 2020-01-11
Title Non-Parametric Trend Tests and Change-Point Detection
Depends R (>= 3.0)
Description The analysis of environmental data often requires
the detection of trends and change-points.
This package includes tests for trend detection
(Cox-Stuart Trend Test, Mann-Kendall Trend Test,
(correlated) Hirsch-Slack Test,
partial Mann-Kendall Trend Test, multivariate (multisite)
Mann-Kendall Trend Test, (Seasonal) Sen’s slope,
partial Pearson and Spearman correlation trend test),
change-point detection (Lanzante's test procedures,
Pettitt's test, Buishand Range Test,
Buishand U Test, Standard Normal Homogeneity Test),
detection of non-randomness (Wallis-Moore Phase Frequency Test,
Bartels rank von Neumann's ratio test, Wald-Wolfowitz Test)
and the two sample Robust Rank-Order Distributional Test.
Imports extraDistr (>= 1.8.0)
Classification/MSC-2010 62F03, 62G10, 62M10, 62P12
License GPL-3
Encoding UTF-8
ZipData yes
NeedsCompilation yes
RoxygenNote 7.0.2
Author Thorsten Pohlert [aut, cre] (https://orcid.org/0000-0003-3855-3025)
Maintainer Thorsten Pohlert <thorsten.pohlert@gmx.de>
Repository CRAN
Date/Publication 2020-01-13 09:40:02 UTC


**R topics documented:**

- `bartels.test` .................................................. 2
- `br.test` .......................................................... 4
- `bu.test` .......................................................... 6
- `cs.test` .......................................................... 8
- `csmk.test` ...................................................... 9
- `hcb` ............................................................. 10
- `lanzante.test` .................................................. 11
- `maxau` ............................................................ 13
- `mk.test` .......................................................... 14
- `mult.mk.test` .................................................... 16
- `PagesData` ........................................................ 18
- `partial.cor.trend.test` ......................................... 19
- `partial.mk.test` .................................................. 20
- `pettitt.test` ..................................................... 22
- `plot.cptest` ..................................................... 23
- `rrod.test` ......................................................... 24
- `sea.sens.slope` .................................................. 26
- `sens.slope` ........................................................ 27
- `smk.test` .......................................................... 28
- `snh.test` .......................................................... 30
- `summary.smktest` ............................................... 32
- `wm.test` .......................................................... 32
- `ww.test` .......................................................... 34

**Index**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bartels.test</code></td>
<td>2</td>
</tr>
<tr>
<td><code>br.test</code></td>
<td>4</td>
</tr>
<tr>
<td><code>bu.test</code></td>
<td>6</td>
</tr>
<tr>
<td><code>cs.test</code></td>
<td>8</td>
</tr>
<tr>
<td><code>csmk.test</code></td>
<td>9</td>
</tr>
<tr>
<td><code>hcb</code></td>
<td>10</td>
</tr>
<tr>
<td><code>lanzante.test</code></td>
<td>11</td>
</tr>
<tr>
<td><code>maxau</code></td>
<td>13</td>
</tr>
<tr>
<td><code>mk.test</code></td>
<td>14</td>
</tr>
<tr>
<td><code>mult.mk.test</code></td>
<td>16</td>
</tr>
<tr>
<td><code>PagesData</code></td>
<td>18</td>
</tr>
<tr>
<td><code>partial.cor.trend.test</code></td>
<td>19</td>
</tr>
<tr>
<td><code>partial.mk.test</code></td>
<td>20</td>
</tr>
<tr>
<td><code>pettitt.test</code></td>
<td>22</td>
</tr>
<tr>
<td><code>plot.cptest</code></td>
<td>23</td>
</tr>
<tr>
<td><code>rrod.test</code></td>
<td>24</td>
</tr>
<tr>
<td><code>sea.sens.slope</code></td>
<td>26</td>
</tr>
<tr>
<td><code>sens.slope</code></td>
<td>27</td>
</tr>
<tr>
<td><code>smk.test</code></td>
<td>28</td>
</tr>
<tr>
<td><code>snh.test</code></td>
<td>30</td>
</tr>
<tr>
<td><code>summary.smktest</code></td>
<td>32</td>
</tr>
<tr>
<td><code>wm.test</code></td>
<td>32</td>
</tr>
<tr>
<td><code>ww.test</code></td>
<td>34</td>
</tr>
</tbody>
</table>

---

**Description**

Performes a rank version of von Neumann’s ratio test as proposed by Bartels. The null hypothesis of randomness is tested against the alternative hypothesis.

**Usage**

`bartels.test(x)`

**Arguments**

- `x` a vector of class "numeric" or a time series object of class "ts"
Details

In this function, the test is implemented as given by Bartels (1982), where the ranks $r_1, \ldots, r_n$ of the $X_1, \ldots, X_n$ are used for the statistic:

$$ T = \frac{\sum_{i=1}^{n} (r_i - r_{i+1})^2}{\sum_{i=1}^{n} (r_i - \bar{r})^2} $$

As proposed by Bartels (1982), the $p$-value is calculated for sample sizes in the range of $10 \leq n < 100$ with the non-standard beta distribution for the range $0 \leq x \leq 4$ with parameters:

$$ a = b = \frac{5n(n + 1)(n - 1)^2}{2(n - 2)(5n^2 - 2n - 9)} - \frac{1}{2} $$

For sample sizes $n \geq 100$ a normal approximation with $N(2, 20/(5n + 7))$ is used for $p$-value calculation.

Value

A list with class "htest"

- `data.name` character string that denotes the input data
- `p.value` the p-value
- `statistic` the test statistic
- `alternative` the alternative hypothesis
- `method` character string that denotes the test

Note

The current function is for complete observations only.

References


See Also

`ww.test`, `wm.test`

Examples

# Example from Schoenwiese (1992, p. 113)
## Number of frost days in April at Munich from 1957 to 1968
##
frost <- ts(data=c(9,12,4,3,0,4,2,1,4,2,9,7), start=1957)
bartels.test(frost)

## Example from Sachs (1997, p. 486)
```r
x <- c(5, 6, 2, 3, 5, 6, 4, 3, 7, 8, 9, 7, 5, 3, 4, 7, 3, 5, 6, 7, 8, 9)
bartels.test(x)
```

```r
## Example from Bartels (1982, p. 43)
x <- c(4, 7, 16, 14, 12, 3, 9, 13, 15, 10, 6, 5, 8, 2, 1, 11, 18, 17)
bartels.test(x)
```

---

**br.test**

**Buishand Range Test for Change-Point Detection**

**Description**
Performes the Buishand range test for change-point detection of a normal variate.

**Usage**

```r
br.test(x, m = 20000)
```

**Arguments**

- `x`: a vector of class "numeric" or a time series object of class "ts"
- `m`: numeric, number of Monte-Carlo replicates, defaults to 20000

**Details**
Let \( X \) denote a normal random variate, then the following model with a single shift (change-point) can be proposed:

\[
x_i = \begin{cases} 
\mu + \epsilon_i & i = 1, \ldots, m \\
\mu + \Delta + \epsilon_i & i = m + 1, \ldots, n 
\end{cases}
\]

with \( \epsilon \approx N(0, \sigma) \). The null hypothesis \( \Delta = 0 \) is tested against the alternative \( \Delta \neq 0 \).

In the Buishand range test, the rescaled adjusted partial sums are calculated as

\[
S_k = \sum_{i=1}^{k} (x_i - \hat{x}) \quad (1 \leq i \leq n)
\]

The test statistic is calculated as:

\[
Rb = \frac{(\max S_k - \min S_k)}{\sigma}
\]

The `p.value` is estimated with a Monte Carlo simulation using `m` replicates.

Critical values based on \( m = 19999 \) Monte Carlo simulations are tabulated for \( Rb/\sqrt{n} \) by Buishand (1982).
Value

A list with class "htest" and "ctest"

data.name character string that denotes the input data
p.value the p-value
statistic the test statistic
null.value the null hypothesis
estimates the time of the probable change point
alternative the alternative hypothesis
method character string that denotes the test
data numeric vector of Sk for plotting

Note

The current function is for complete observations only.

References


See Also

*efp sctest.efp*

Examples

```r
data(Nile)
(out <- br.test(Nile))
plot(out)

data(PagesData) ; br.test(PagesData)
```
Buishand U Test for Change-Point Detection

Description

Performes the Buishand U test for change-point detection of a normal variate.

Usage

bu.test(x, m = 20000)

Arguments

x

a vector of class "numeric" or a time series object of class "ts"

m

numeric, number of Monte-Carlo replicates, defaults to 20000

Details

Let $X$ denote a normal random variate, then the following model with a single shift (change-point) can be proposed:

$$x_i = \begin{cases} 
\mu + \epsilon_i, & i = 1, \ldots, m \\
\mu + \Delta + \epsilon_i & i = m + 1, \ldots, n
\end{cases}$$

with $\epsilon \approx N(0, \sigma)$. The null hypothesis $\Delta = 0$ is tested against the alternative $\Delta \neq 0$.

In the Buishand U test, the rescaled adjusted partial sums are calculated as

$$S_k = \sum_{i=1}^{k} (x_i - \bar{x}) \quad (1 \leq i \leq n)$$

The sample standard deviation is

$$D_x = \sqrt{\frac{n}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})}$$

The test statistic is calculated as:

$$U = \left[ n(n+1) \right]^{-1} \sum_{k=1}^{n-1} (S_k/D_x)^2$$

The p.value is estimated with a Monte Carlo simulation using $m$ replicates.

Critical values based on $m = 19999$ Monte Carlo simulations are tabulated for $U$ by Buishand (1982, 1984).
Value

A list with class "htest" and "cptest"

- `data.name`: character string that denotes the input data
- `p.value`: the p-value
- `statistic`: the test statistic
- `null.value`: the null hypothesis
- `estimates`: the time of the probable change point
- `alternative`: the alternative hypothesis
- `method`: character string that denotes the test
- `data`: numeric vector of Sk for plotting

Note

The current function is for complete observations only.

References


See Also

`efp sctest.efp`

Examples

```r
data(Nile)
(out <- bu.test(Nile))
plot(out)

data(PagesData)
bu.test(PagesData)
```
Description

Performs the non-parametric Cox and Stuart trend test

Usage

cs.test(x)

Arguments

x a vector or a time series object of class "ts"

Details

First, the series is divided by three. It is compared, whether the data of the first third of the series are larger or smaller than the data of the last third of the series. The test statistic of the Cox-Stuart trend test for $n > 30$ is calculated as:

$$z = \frac{|S - \frac{n}{6}|}{\sqrt{\frac{n}{12}}}$$

where $S$ denotes the maximum of the number of signs, i.e. + or −, respectively. The $z$-statistic is normally distributed. For $n \leq 30$ a continuity correction of $-0.5$ is included in the denominator.

Value

An object of class "htest"

method a character string indicating the chosen test
data.name a character string giving the name(s) of the data
statistic the Cox-Stuart z-value
alternative a character string describing the alternative hypothesis
p.value the p-value for the test

Note

NA values are omitted. Many ties in the series will lead to reject H0 in the present test.

References

csmk.test

See Also

mk.test

Examples

## Example from Schoenwiese (1992, p. 114)
## Number of frost days in April at Munich from 1957 to 1968
## z = -0.5, Accept H0
frost <- ts(data=c(9,12,4,3,0,4,2,1,4,2,9,7), start=1957)
cs.test(frost)

## Example from Sachs (1997, p. 486-487)
## z ~ 2.1, Reject H0 on a level of p = 0.0357
x <- c(5,6,2,3,5,6,4,3,7,8,9,7,5,3,4,7,3,5,6,7,8,9)
cs.test(x)
cs.test(Nile)

---

csmk.test  Correlated Seasonal Mann-Kendall Test

Description

Performs a Seasonal Mann-Kendall test under the presence of correlated seasons.

Usage

csmk.test(x, alternative = c("two.sided", "greater", "less"))

Arguments

x a time series object with class ts comprising >= 2 seasons; NA values are not allowed
alternative the alternative hypothesis, defaults to two.sided

Details

The Mann-Kendall scores are first computed for each season separately. The variance - covariance matrix is computed according to Libiseller and Grimvall (2002). Finally the corrected Z-statistics for the entire series is calculated as follows, whereas a continuity correction is employed for \( n \leq 10 \):

\[
z = \frac{1^T S}{\sqrt{1^T \Gamma 1}}
\]

where

\( z \) denotes the quantile of the normal distribution, \( 1 \) indicates a vector with all elements equal to one, \( S \) is the vector of Mann-Kendall scores for each season and \( \Gamma \) denotes the variance - covariance matrix.
Value

An object with class "htest"

- `data.name` character string that denotes the input data
- `p.value` the p-value for the entire series
- `statistic` the z quantile of the standard normal distribution for the entire series
- `null.value` the null hypothesis
- `estimates` the estimates S and varS for the entire series
- `alternative` the alternative hypothesis
- `method` character string that denotes the test
- `cov` the variance-covariance matrix

Note

Ties are not corrected. Current Version is for complete observations only.

References


See Also

- `cor`, `cor.test`, `mk.test`, `smk.test`

Examples

```r
csmk.test(nottem)
```

---

**hcb**

*Monthly concentration of particle bound HCB, River Rhine*

Description

Time series of monthly concentration of particle bound Hexachlorobenzene (HCB) in µg/kg at six different monitoring sites at the River Rhine, 1995.1-2006.12

Usage

```r
data(hcb)
```
Format

a time series object of class "mts"

- we first column, series of station Weil (RKM 164.3)
- ka second column, series of station Karlsruhe-Iffezheim (RKM 333.9)
- mz third column, series of station Mainz (RKM 498.5)
- ko fourth column, series of station Koblenz (RKM 590.3)
- bh fifth column, series of station Bad Honnef (RKM 645.8)
- bi sixth column, series of station Bimmen (RKM 865.0)

Details

NO DATA values in the series were filled with estimated values using linear interpolation (see approx).

The Rhine Kilometer (RKM) is in increasing order from source to mouth of the River Rhine.

Source


References


Examples

data(hcb)
plot(hcb)
mult.mk.test(hcb)

lanzante.test  Lanzante’s Test for Change Point Detection

Description

Performes a non-parametric test after Lanzante in order to test for a shift in the central tendency of a time series. The null hypothesis, no shift, is tested against the alternative, shift.

Usage

lanzante.test(x, method = c("wilcox.test", "rrod.test"))
Arguments

- **x**: a vector of class "numeric" or a time series object of class "ts"
- **method**: the test method. Defaults to "wilcox.test".

Details

Let $X$ denote a continuous random variable, then the following model with a single shift (change-point) can be proposed:

$$x_i = \begin{cases} 
\theta + \epsilon_i, & i = 1, \ldots, m \\
\theta + \Delta + \epsilon_i & i = m + 1, \ldots, n
\end{cases}$$

with $\theta(\epsilon) = 0$. The null hypothesis, $H: \Delta = 0$ is tested against the alternative $A: \Delta \neq 0$.

First, the data are transformed into increasing ranks and for each time-step the adjusted rank sum is computed:

$$U_k = 2 \sum_{i=1}^{k} r_i - k (n + 1)$$

$k = 1, \ldots, n$

The probable change point is located at the absolute maximum of the statistic:

$$m = k(\max |U_k|)$$

For **method** = "wilcox.test" the Wilcoxon-Mann-Whitney two-sample test is performed, using $m$ to split the series. Otherwise, the robust rank-order distributional test (**rrod.test** is performed.

Value

A list with class "htest" and "cptest".

References


See Also

- **pettitt.test**
### maxau

**Annual suspended sediment concentration and flow data, River Rhine**

### Description

Annual time series of average suspended sediment concentration (s) in mg/l and average discharge (Q) in m³/s at the River Rhine, 1965.1-2009.1

### Usage

```r
data(maxau)
```

### Format

A time series object of class "mts"

- **s.** First column, suspended sediment concentration
- **Q.** Second column, average discharge

### Source

Bundesanstalt für Gewässerkunde, Koblenz, Deutschland (Federal Institute of Hydrology, Koblenz, Germany)

### Examples

```r
data(maxau)
plot(maxau)
```
Description

Performs the Mann-Kendall Trend Test

Usage

```r
mk.test(x, alternative = c("two.sided", "greater", "less"), continuity = TRUE)
```

Arguments

- `x`: a vector of class "numeric" or a time series object of class "ts"
- `alternative`: the alternative hypothesis, defaults to "two.sided"
- `continuity`: logical, indicates whether a continuity correction should be applied, defaults to TRUE.

Details

The null hypothesis is that the data come from a population with independent realizations and are identically distributed. For the two sided test, the alternative hypothesis is that the data follow a monotonic trend. The Mann-Kendall test statistic is calculated according to:

\[
S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} \text{sgn}(x_j - x_k)
\]

with \text{sgn} the signum function (see `sign`).

The mean of \( S \) is \( \mu = 0 \). The variance including the correction term for ties is

\[
\sigma^2 = \left\{ \frac{n(n-1)(2n+5) - \sum_{j=1}^{p} t_j (t_j - 1) (2t_j + 5)}{18} \right\}
\]

where \( p \) is the number of the tied groups in the data set and \( t_j \) is the number of data points in the \( j \)-th tied group. The statistic \( S \) is approximately normally distributed, with

\[
z = \frac{S}{\sigma}
\]

If `continuity = TRUE` then a continuity correction will be employed:

\[
z = \text{sgn}(S) \frac{|S| - 1}{\sigma}
\]

The statistic \( S \) is closely related to Kendall's \( \tau \):

\[
\tau = S/D
\]
where
\[
D = \left[ \frac{1}{2} n (n - 1) - \frac{1}{2} \sum_{j=1}^{p} t_j (t_j - 1) \right]^{1/2} \left[ \frac{1}{2} n (n - 1) \right]^{1/2}
\]

Value

A list with class "htest"

- data.name: character string that denotes the input data
- p.value: the p-value
- statistic: the \(z\) quantile of the standard normal distribution
- null.value: the null hypothesis
- estimates: the estimates \(S\), \(\text{var}S\) and \(\tau\)
- alternative: the alternative hypothesis
- method: character string that denotes the test

Note

Current Version is for complete observations only.

References


See Also

cor.test, MannKendall, partial.mk.test, sens.slope

Examples

data(Nile)
mk.test(Nile, continuity = TRUE)

```r
##
n <- length(Nile)
cor.test(x=(1:n), y=Nile, meth="kendall", continuity = TRUE)
```
Description

Performs a Multivariate (Multisite) Mann-Kendall test.

Usage

mult.mk.test(x, alternative = c("two.sided", "greater", "less"))

Arguments

x a time series object of class "ts"
alternative the alternative hypothesis, defaults to two.sided

Details

The Mann-Kendall scores are first computed for each variate (side) separately.

\[ S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} \text{sgn} (x_j - x_k) \]

with \( \text{sgn} \) the signum function (see \texttt{sign}).

The variance - covariance matrix is computed according to Libiseller and Grimvall (2002).

\[ \Gamma_{xy} = \frac{1}{3} \left[ K + 4 \sum_{j=1}^{n} R_{jx} R_{jy} - n (n + 1) (n + 1) \right] \]

with

\[ K = \sum_{1 \leq i < j \leq n} \text{sgn} \{ (x_j - x_i)(y_j - y_i) \} \]

and

\[ R_{jx} = \left\{ n + 1 + \sum_{i=1}^{n} \text{sgn} (x_j - x_i) \right\} / 2 \]

Finally, the corrected \( z \)-statistics for the entire series is calculated as follows, whereas a continuity correction is employed for \( n \leq 10 \):

\[ z = \frac{\sum_{i=1}^{d} S_i}{\sqrt{\sum_{j=1}^{d} \sum_{i=1}^{d} \Gamma_{ij}}} \]

where

\( z \) denotes the quantile of the normal distribution \( S \) is the vector of Mann-Kendall scores for each variate (site) \( 1 \leq i \leq d \) and \( \Gamma \) denotes symmetric variance - covariance matrix.
mult.mk.test

Value

An object with class "htest"

data.name character string that denotes the input data
p.value the p-value for the entire series
statistic the z quantile of the standard normal distribution for the entire series
null.value the null hypothesis
estimates the estimates S and varS for the entire series
alternative the alternative hypothesis
method character string that denotes the test
cov the variance - covariance matrix

Note

Ties are not corrected. Current Version is for complete observations only.

References


See Also

cor, cor.test, mk.test, smk.test

Examples

data(hcb)
mult.mk.test(hcb)
PagesData

Simulated data of Page (1955) as test-example for change-point detection

Description

Simulated data of Page (1955) as test-example for change-point detection taken from Table 1 of Pettitt (1979)

Usage

data(PagesData)

Format

a vector that contains 40 elements

Details

According to the publication of Pettitt (1979), the series comprise a significant $p = 0.014$ change-point at $i = 17$. The function `pettitt.test` computes the same U statistics as given by Pettitt (1979) in Table 1, row 4.

References

Page, E. S. (1954), A test for a change in a parameter occurring at an unknown point. *Biometrika* 41, 100–114.


See Also

`pettitt.test`

Examples

data(PagesData)
pettitt.test(PagesData)
partial.cor.trend.test

*Partial Correlation Trend Test*

**Description**

Performs a partial correlation trend test with either Pearson’s or Spearman’s correlation coefficients \((r(t_x.z))\).

**Usage**

```r
partial.cor.trend.test(x, z, method = c("pearson", "spearman"))
```

**Arguments**

- `x`: a "vector" or "ts" object that contains the variable, which is tested for trend (i.e. correlated with time)
- `z`: a "vector" or "ts" object that contains the co-variate, which will be partialled out
- `method`: a character string indicating which correlation coefficient is to be computed. One of "pearson" (default) or "spearman", can be abbreviated.

**Details**

This function performs a partial correlation trend test using either the "pearson" correlation coefficient, or the "spearman" rank correlation coefficient (Hipel and McLoed (1994), p. 882). The partial correlation coefficient for the response variable "x" with time "t", when the effect of the explanatory variable "z" is partialled out, is defined as:

\[
    r_{t_x.z} = \frac{r_{t_x} - r_{t_z} r_{x_z}}{\sqrt{1 - r_{t_z}^2} \sqrt{1 - r_{x_z}^2}}
\]

The H0: \( r_{t_x.z} = 0 \) (i.e. no trend for "x", when effect of "z" is partialled out) is tested against the alternate Hypothesis, that there is a trend for "x", when the effect of "z" is partialled out.

The partial correlation coefficient is tested for significance with the student t distribution on \( df = n - 2 \) degree of freedom.

**Value**

An object of class "htest"

- `method`: a character string indicating the chosen test
- `data.name`: a character string giving the name(s) of the data
- `statistic`: the value of the test statistic
- `estimate`: the partial correlation coefficient \(r(t_x.z)\)
- `parameter`: the degrees of freedom of the test statistic in the case that it follows a t distribution
alternative               a character string describing the alternative hypothesis
p.value                   the p-value of the test
null.value                The value of the null hypothesis

Note
Current Version is for complete observations only.

References

See Also
cor, cor.test, partial.r, partial.mk.test,

Examples
data(maxau)
a <- tsp(maxau); tt <- a[1]:a[2]
s <- maxau[,"s"] ; Q <- maxau[,"Q"]
maxau.df <- data.frame(Year = tt, s = s, Q = Q)
plot(maxau.df)

partial.cor.trend.test(s, Q, method="pearson")
partial.cor.trend.test(s, Q, method="spearman")

---

partial.mk.test Partial Mann-Kendall Trend Test

Description
Performs a partial Mann-Kendall Trend Test

Usage
partial.mk.test(x, y, alternative = c("two.sided", "greater", "less"))

Arguments
x               a "vector" or "ts" object that contains the variable, which is tested for trend (i.e. correlated with time)
y               a "vector" or "ts" object that contains the variable, which effect on "x" is partialled out
alternative    character, the alternative method; defaults to "two.sided"
Details

According to Libiseller and Grimvall (2002), the test statistic for $x$ with its covariate $y$ is

$$z = \frac{S_x - r_{xy}S_y}{\left[(1 - r_{xy}^2) n (n - 1) (2n + 5) / 18\right]^{0.5}}$$

where the correlation $r$ is calculated as:

$$r_{xy} = \frac{\sigma_{xy}}{n(n - 1)(2n + 5)/18}$$

The conditional covariance between $x$ and $y$ is

$$\sigma_{xy} = \frac{1}{3} \left[ K + 4 \sum_{j=1}^{n} R_{jx}R_{jy} - n(n + 1)(n + 1) \right]$$

with

$$K = \sum_{1 \leq i < j \leq n} \text{sgn}\left\{(x_j - x_i)(y_j - y_i)\right\}$$

and

$$R_{jx} = \left\{ n + 1 + \sum_{i=1}^{n} \text{sgn}(x_j - x_i) \right\} / 2$$

Value

A list with class "htest"

- **method**: a character string indicating the chosen test
- **data.name**: a character string giving the name(s) of the data
- **statistic**: the value of the test statistic
- **estimate**: the Mann-Kendall score $S$, the variance var$S$ and the correlation between $x$ and $y$
- **alternative**: a character string describing the alternative hypothesis
- **p.value**: the p-value of the test
- **null.value**: the null hypothesis

Note

Current Version is for complete observations only. The test statistic is not corrected for ties.
References


See Also

partial.cor.trend.test,

Examples

data(maxau)
s <- maxau[,"s"]
Q <- maxau[,"Q"]
partial.mk.test(s,Q)

pettitt.test

Pettitt’s Test for Change-Point Detection

Description

Performs a non-parametric test after Pettitt in order to test for a shift in the central tendency of a time series. The H0-hypothesis, no change, is tested against the HA-Hypothesis, change.

Usage

pettitt.test(x)

Arguments

x

a vector of class "numeric" or a time series object of class "ts"

Details

In this function, the test is implemented as given by Verstraeten et. al. (2006), where the ranks 
r_1,\ldots,r_n of the X_i,\ldots,X_n are used for the statistic:

\[ U_k = 2 \sum_{i=1}^{k} r_i - k(n + 1) \quad k = 1,\ldots,n \]

The test statistic is the maximum of the absolute value of the vector:

\[ \hat{U} = \max |U_k| \]

The probable change-point K is located where \( \hat{U} \) has its maximum. The approximate probability for a two-sided test is calculated according to

\[ p = 2 \exp\left( -6K^2/(T^2 + T^2) \right) \]
Value

A list with class "htest" and "cptest"

Note

The current function is for complete observations only. The approximate probability is good for $p \leq 0.5$.

References


See Also

`efp sctest.efp`

Examples

data(maxau) ; plot(maxau[,"s"])
s.res <- pettitt.test(maxau[,"s"])
n <- s.res$nobs
i <- s.res$estimate
s.1 <- mean(maxau[1:i,"s"])
s.2 <- mean(maxau[(i+1):n,"s"])
s <- ts(c(rep(s.1,i), rep(s.2,(n-i))))
tsp(s) <- tsp(maxau[,"s"])
lines(s, lty=2)
print(s.res)

data(PagesData) ; pettitt.test(PagesData)
rrod.test

Robust Rank-Order Distributional Test

Description
Performs Fligner-Pollicello robust rank-order distributional test for location.

Usage
rrod.test(x, ...)

## Default S3 method:
rrod.test(x, y, alternative = c("two.sided", "less", "greater"), ...)

## S3 method for class 'formula'
rrod.test(formula, data, subset, na.action, ...)

Arguments
x a vector of data values.
... further arguments to be passed to or from methods.
y an optional numeric vector of data values.
alternative the alternative hypothesis. Defaults to "two.sided".
formula a formula of the form response ~ group where response gives the data values and group a vector or factor of the corresponding groups.
data an optional matrix or data frame (or similar: see model.frame) containing the variables in the formula formula. By default the variables are taken from environment(formula).
subset an optional vector specifying a subset of observations to be used.
na.action a function which indicates what should happen when the data contain NAs. Defaults to getOption("na.action").
Details

The non-parametric RROD two-sample test can be used to test for differences in location, whereas it does not assume variance homogeneity.

Let $X$ and $Y$ denote two samples with sizes $n_x$ and $n_y$ of a continuous variable. First, the combined sample is transformed into ranks in increasing order. Let $S_{xi}$ and $S_{yj}$ denote the counts of $Y$ ($X$) values having a lower rank than $x_i$ ($y_j$). The mean counts are:

$$
\bar{S}_x = \frac{\sum_{i=1}^{n_x} S_{xi}}{n_x}
$$

$$
\bar{S}_y = \frac{\sum_{j=1}^{n_y} S_{yj}}{n_y}
$$

The variances are:

$$
s^2_{Sx} = \frac{\sum_{i=1}^{n_x} (S_{xi} - \bar{S}_x)^2}{n_x}
$$

$$
s^2_{Sy} = \frac{\sum_{j=1}^{n_y} (S_{yj} - \bar{S}_y)^2}{n_y}
$$

The test statistic is:

$$
z = \frac{1}{2} \left( \frac{n_x \bar{S}_x - n_y \bar{S}_y}{\bar{S}_x \bar{S}_y + s^2_{Sx} + s^2_{Sy}} \right)^{1/2}
$$

The two samples have significantly different location parameters, if $|z| > z_{1-\alpha/2}$. The function calculates the $p$-values of the null hypothesis for the selected alternative than can be "two.sided", "greater" or "less".

Value

A list with class "htest".

References


See Also

wilcox.test

Examples

## Two-sample test.
## Hollander & Wolfe (1973), 69f.
## Permeability constants of the human chorioamnion (a placental
## membrane) at term (x) and between 12 to 26 weeks gestational
## age (y). The alternative of interest is greater permeability
## of the human chorioamnion for the term pregnancy.
## x <- c(0.80, 0.83, 1.89, 1.04, 1.45, 1.38, 1.91, 1.64, 0.73, 1.46)
y <- c(1.15, 0.88, 0.90, 0.74, 1.21)
rrod.test(x, y, alternative = "g")

## Formula interface.
boxplot(Ozone ~ Month, data = airquality)
rrod.test(Ozone ~ Month, data = airquality,
subset = Month %in% c(5, 8))

Description

Computes seasonal Sen’s slope for linear rate of change

Usage

sea.sens.slope(x)

Arguments

x a time series object of class "ts"

Details

According to Hirsch et al. (1982) the seasonal Sen’s slope is calculated as follows:

\[ d_{ijk} = \frac{x_{ij} - x_{ik}}{j - k} \]

for each \((x_{ij}, x_{ik})\) pair \(i = 1, \ldots, m\), where \(1 \leq k < j \leq n_i\) and \(n_i\) is the number of known values in the \(i\)th season. The seasonal slope estimator is the median of the \(d_{ijk}\) values.

Value

numeric, Seasonal Sen’s slope.
Note
Current Version is for complete observations only.

References

See Also
smk.test,

Examples
`sea.sens.slope(nottem)`

<table>
<thead>
<tr>
<th>sens.slope</th>
<th>Sen’s slope</th>
</tr>
</thead>
</table>

Description
Computes Sen’s slope for linear rate of change and corresponding confidence intervals.

Usage
`sens.slope(x, conf.level = 0.95)`

Arguments

<table>
<thead>
<tr>
<th>x</th>
<th>numeric vector or a time series object of class &quot;ts&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>conf.level</td>
<td>numeric, the level of significance</td>
</tr>
</tbody>
</table>

Details
This test computes both the slope (i.e. linear rate of change) and confidence levels according to Sen’s method. First, a set of linear slopes is calculated as follows:

\[ d_k = \frac{x_j - x_i}{j - i} \]

for \( 1 \leq i < j \leq n \), where d is the slope, x denotes the variable, n is the number of data, and i, j are indices.

Sen’s slope is then calculated as the median from all slopes: \( b_{Sen} = \text{median}(d_k) \).
This function also computes the upper and lower confidence limits for sens slope.
Value

A list of class "htest".

- `estimates` numeric, Sen's slope
- `data.name` character string that denotes the input data
- `p.value` the p-value
- `statistic` the z quantile of the standard normal distribution
- `null.value` the null hypothesis
- `conf.int` upper and lower confidence limit
- `alternative` the alternative hypothesis
- `method` character string that denotes the test

Note

Current Version is for complete observations only.

References


Examples

data(maxau)
sens.slope(maxau[,"s"])
smk.test(maxau[,"s"])
Arguments

- **x**: a time series object with class `ts` comprising >= 2 seasons; NA values are not allowed.
- **alternative**: the alternative hypothesis, defaults to `two.sided`.
- **continuity**: logical, indicates whether a continuity correction should be done; defaults to `TRUE`.

Details

The Mann-Kendall statistic for the $g$-th season is calculated as:

$$S_g = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \text{sgn}(x_{jg} - x_{ig}), \quad (1 \leq g \leq m)$$

with `sgn` the signum function (see `sign`).

The mean of $S_g$ is $\mu_g = 0$. The variance including the correction term for ties is

$$\sigma^2_g = \left\{ n (n-1) (2n+5) - \sum_{j=1}^{p} t_{jg} (t_{jg} - 1) (2t_{jg} + 5) \right\} / 18 \quad (1 \leq g \leq m)$$

The seasonal Mann-Kendall statistic for the entire series is calculated according to

$$\hat{S} = \sum_{g=1}^{m} S_g \quad \hat{\sigma}^2 = \sum_{g=1}^{m} \sigma^2_g$$

The statistic $S_g$ is approximately normally distributed, with

$$z_g = S_g / \sigma_g$$

If `continuity = TRUE` then a continuity correction will be employed:

$$z = \text{sgn}(S_g) \ (|S_g| - 1) / \sigma_g$$

Value

An object with class "htest" and "smktest"

- **data.name**: character string that denotes the input data
- **p.value**: the p-value for the entire series
- **statistic**: the z quantile of the standard normal distribution for the entire series
- **null.value**: the null hypothesis
- **estimates**: the estimates $S$ and $\text{var}S$ for the entire series
- **alternative**: the alternative hypothesis
- **method**: character string that denotes the test
- **Sg**: numeric vector that contains $S$ scores for each season
- **varSg**: numeric vector that contains $\text{var}S$ for each season
- **pvalg**: numeric vector that contains p-values for each season
- **Zg**: numeric vector that contains z-quantiles for each season
References


Examples

```r
res <- snh.test(nottem)
## print method
res
## summary method
summary(res)
```

---

**snh.test**

*Standard Normal Homogeneity Test (SNHT) for Change-Point Detection*

**Description**

Performs the Standard Normal Homogeneity Test (SNHT) for change-point detection of a normal variate.

**Usage**

```r
snh.test(x, m = 20000)
```

**Arguments**

- `x` a vector of class "numeric" or a time series object of class "ts"
- `m` numeric, number of Monte-Carlo replicates, defaults to 20000

**Details**

Let $X$ denote a normal random variate, then the following model with a single shift (change-point) can be proposed:

$$
x_i = \begin{cases} 
\mu + \epsilon_i, & i = 1, \ldots, m \\
\mu + \Delta + \epsilon_i & i = m + 1, \ldots, n
\end{cases}
$$

with $\epsilon \approx N(0, \sigma)$. The null hypothesis $\Delta = 0$ is tested against the alternative $\Delta \neq 0$.

The test statistic for the SNHT test is calculated as follows:
\[ T_k = k z_1^2 + (n - k) z_2^2 \quad (1 \leq k < n) \]

where

\[ z_1 = \frac{1}{k} \sum_{i=1}^{k} \frac{x_i - \bar{x}}{\sigma} \quad z_2 = \frac{1}{n-k} \sum_{i=k+1}^{n} \frac{x_i - \bar{x}}{\sigma}. \]

The critical value is:

\[ T = \max T_k. \]

The p.value is estimated with a Monte Carlo simulation using \( m \) replicates.

Critical values based on \( m = 1,000,000 \) Monte Carlo simulations are tabulated for \( T \) by Khaliq and Ouarda (2007).

Value

A list with class "htest" and "cptest"

- data.name: character string that denotes the input data
- p.value: the p-value
- statistic: the test statistic
- null.value: the null hypothesis
- estimates: the time of the probable change point
- alternative: the alternative hypothesis
- method: character string that denotes the test
- data: numeric vector of \( T_k \) for plotting

Note

The current function is for complete observations only.

References


See Also

- efp sctest.efp
Examples

```r
data(Nile)
(out <- snh.test(Nile))
plot(out)

data(PagesData) ; snh.test(PagesData)
```

---

**summary.smktest**  
*Object summaries*

**Description**

Generic function "summary" for objects of class smktest.

**Usage**

```r
## S3 method for class 'smktest'
summary(object, ...)
```

**Arguments**

- `object`: an object of class `smktest`
- `...`: further arguments, currently ignored

---

**wm.test**  
*Wallis and Moore Phase-Frequency Test*

**Description**

Performs the non-parametric Wallis and Moore phase-frequency test for testing the H0-hypothesis, whether the series comprises random data, against the HA-Hypothesis, that the series is significantly different from randomness (two-sided test).

**Usage**

```r
wm.test(x)
```

**Arguments**

- `x`: a vector or a time series object of class "ts"
Details

The test statistic of the phase-frequency test for \( n > 30 \) is calculated as:

\[
    z = \left| \frac{h - \frac{2n-7}{3}}{\sqrt{\frac{16n-29}{90}}} \right|
\]

where \( h \) denotes the number of phases, whereas the first and the last phase is not accounted. The \( z \)-statistic is normally distributed. For \( n \leq 30 \) a continuity correction of \(-0.5\) is included in the denominator.

Value

An object of class "htest"

- method: a character string indicating the chosen test
- data.name: a character string giving the name(s) of the data
- statistic: the Wallis and Moore z-value
- alternative: a character string describing the alternative hypothesis
- p.value: the p-value for the test

Note

NA values are omitted. Many ties in the series will lead to reject \( H_0 \) in the present test.

References


See Also

- mk.test

Examples

```r
## Example from Schoenwiese (1992, p. 113)
## Number of frost days in April at Munich from 1957 to 1968
## \( z = -0.124 \), Accept \( H_0 \)
frost <- ts(data=c(9,12,4,3,0,4,2,1,4,2,9,7), start=1957)
wm.test(frost)

## Example from Sachs (1997, p. 486)
## \( z = 2.56 \), Reject \( H_0 \) on a level of \( p < 0.05 \)
x <- c(5,6,2,3,5,6,4,3,7,8,9,7,5,3,4,7,3,5,6,7,8,9)
wm.test(x)
```
ww.test

Wald-Wolffowitz Test for Independence and Stationarity

Description
Performs the non-parametric Wald-Wolffowitz test for independence and stationarity.

Usage
ww.test(x)

Arguments
x a vector or a time series object of class "ts"

Details
Let $x_1, x_2, ..., x_n$ denote the sampled data, then the test statistic of the Wald-Wolffowitz test is calculated as:

$$R = \sum_{i=1}^{n-1} x_i x_{i+1} + x_1 x_n$$

The expected value of $R$ is:

$$E(R) = \frac{s_1^2 - s_2}{n - 1}$$

The expected variance is:

$$V(R) = \frac{s_2^2 - s_4}{n - 1} - E(R)^2 + \frac{s_4^4 - 4s_1^2s_2^2 + 4s_1s_3s_4 + s_2^4 - 2s_4}{(n - 1)(n - 2)}$$

with:

$$s_t = \sum_{i=1}^{n} x_i^t, \quad t = 1, 2, 3, 4$$

For $n > 10$ the test statistic is normally distributed, with:

$$z = \frac{R - E(R)}{\sqrt{V(R)}}$$

ww.test calculates p-values from the standard normal distribution for the two-sided case.
Value

An object of class "htest"

- `method`: a character string indicating the chosen test
- `data.name`: a character string giving the name(s) of the data
- `statistic`: the Wald-Wolfowitz z-value
- `alternative`: a character string describing the alternative hypothesis
- `p.value`: the p-value for the test

Note

NA values are omitted.

References


Examples

```r
ww.test(nottem)
ww.test(Nile)

set.seed(200)
x <- rnorm(100)
ww.test(x)
```
Index

*Topic datasets
  hcb, 10
  maxau, 13
  PagesData, 18

*Topic htest
  bartels.test, 2
  br.test, 4
  bu.test, 6
  cs.test, 8
  lanzante.test, 11
  pettitt.test, 22
  rrod.test, 24
  snh.test, 30
  wm.test, 32
  ww.test, 34

*Topic multivariate
  csmk.test, 9
  mult.mk.test, 16
  partial.cor.trend.test, 19

*Topic nonparametric
  bartels.test, 2
  cs.test, 8
  csmk.test, 9
  lanzante.test, 11
  mk.test, 14
  mult.mk.test, 16
  partial.cor.trend.test, 19
  partial.mk.test, 20
  pettitt.test, 22
  rrod.test, 24
  sea.sens.slope, 26
  sens.slope, 27
  smk.test, 28
  snh.test, 30

*Topic univar
  bartels.test, 2
  br.test, 4
  bu.test, 6
  lanzante.test, 11
  mk.test, 14
  pettitt.test, 22
  sea.sens.slope, 26
  sens.slope, 27
  smk.test, 28
  snh.test, 30

approx, 11

 bartels.test, 2
  br.test, 4
  bu.test, 6
  cor, 10, 17, 20

 cor.test, 10, 15, 17, 20
  cs.test, 8
  csmk.test, 9

 efp, 5, 7, 23, 31

 hcb, 10
 Hirsch-Slack-Test (smk.test), 28
 lanzante.test, 11
Mann-Kendall (mult.mk.test), 16
MannKendall, 15
maxau, 13
mk.test, 9, 10, 14, 17, 33
model.frame, 24
mult.mk.test, 16
Multiside (mult.mk.test), 16

PagesData, 18
partial.cor.trend.test, 19, 22
partial.mk.test, 15, 20, 20
partial.r, 20
pettit.test, 12, 18, 22
plot.cptest, 23
rrod.test, 12, 24

sctest.efp, 5, 7, 23, 31
sea.sens.slope, 26
sens.slope, 15, 27
sign, 14, 16, 29
smk.test, 10, 17, 27, 28
snh.test, 30
summary.smktest, 32

wilcox.test, 26
wm.test, 3, 32
ww.test, 3, 34