Package ‘wwntests’

May 18, 2020

Type Package
Title Hypothesis Tests for Functional Time Series
Version 1.0.1
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Description Provides an array of white noise hypothesis tests for functional data and related visualizations. These include tests based on the norms of autocovariance operators that are built under both strong and weak white noise assumptions. Additionally, tests based on the spectral density operator and on principal component dimensional reduction are included, which are built under strong white noise assumptions. These methods are described in Kokoszka et al. (2017) <doi:10.1016/j.jmva.2017.08.004>, Characiejus and Rice (2019) <doi:10.1016/j.ecosta.2019.01.003>, and Gabrys and Kokoszka (2007) <doi:10.1198/016214507000001111>, respectively.
License GPL-3
Encoding UTF-8
LazyData true
Depends R (>= 3.4.0)
Imports sde, stats, ftsa, rainbow, MASS, graphics
Suggests testthat, knitr, rmarkdown, fOptions, CompQuadForm, tensorA
RoxygenNote 7.0.2
VignetteBuilder knitr
BugReports https://github.com/jimthemadmanlahey/wwntests/issues
NeedsCompilation no
Repository CRAN
Date/Publication 2020-05-18 20:10:02 UTC


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autocorrelation_coeff_h  
Computes the approximate functional autocorrelation coefficient at a given lag.

Description  
Computes the approximate functional autocorrelation coefficient at a given lag.

Usage  
autocorrelation_coeff_h(f_data, lag)
Arguments

- **f_data**: The functional data matrix with observed functions in the columns.
- **lag**: The lag to use to compute the single lag test statistic.

**Value**

numeric value; the approximate functional autocorrelation coefficient at lag h.

---

**Description**

`autocorrelation_coeff_plot` computes the 1-alpha upper confidence bounds for the functional autocorrelation coefficients at lags h = 1:K under both weak white noise (WWN) and strong white noise (SWN) assumptions. It plots the coefficients as well as the bounds for all lags h = 1:K. Note, the SWN bound is constant, while the WWN is dependent on the lag.

**Usage**

```r
autocorrelation_coeff_plot(
  f_data,
  K = 20,
  alpha = 0.05,
  M = NULL,
  low_disc = FALSE,
  wwn_bound = TRUE
)
```

**Arguments**

- **f_data**: The functional data matrix with observed functions in the columns.
- **K**: A positive Integer value. The maximum lag for which to compute the single-lag test (tests will be computed for lags h in 1:K).
- **alpha**: A numeric value between 0 and 1 specifying the significance level to be used in the single-lag test. The default value is 0.05.
- **M**: A positive Integer value. Determines the number of Monte-Carlo simulations employed in the Welch-Satterthwaite approximation of the limiting distribution of the test statistics, for each test.
- **low_disc**: A Boolean value, FALSE by default. If given TRUE, uses low-discrepancy sampling in the Monte-Carlo method. Note, low-discrepancy sampling will yield deterministic results. Requires the ’fOptions’ package.
- **wwn_bound**: A Boolean value allowing the user to turn off the weak white noise bound. TRUE by default. Speeds up computation when FALSE.
Details
This function computes and plots autocorrelation coefficients at lag h, for h in 1:K. It also computes an estimated asymptotic 1 - alpha confidence bound, under the assumption that the series forms a weak white noise. Additionally, it computes a similar (constant) bound under the assumption the series form a strong white noise. Please see the vignette or the references for a more complete treatment.

Value
Plot of the estimated autocorrelation coefficients for lags h in 1:K with the weak white noise 1-alpha upper confidence bound for each lag, as well as the constant strong white noise 1-alpha confidence bound.

References

Examples
b <- brown_motion(75, 40)
autocorrelation_coeff_plot(b)
autocorrelation_coeff_plot(b, M = 200, low_disc = TRUE)

autocov_approx_h

Compute the approximate autocovariance at specified lag

Description
autocov_approx_h Computes the approximate autocovariance for a given lag h of the functional data

Usage
autocov_approx_h(f_data, lag)

Arguments
f_data the functional data matrix with observed functions in the columns
lag the lag to use to compute the single lag test statistic

Value
A 2-dimensional array encoding the autocovariance matrix for a given lag h.
**Bartlett Kernel Function**

**Description**

bartlett_kernel Computes the Bartlett kernel function at a given point value.

**Usage**

`bartlett_kernel(x)`

**Arguments**

- `x`  
  the point value at which the kernel function is evaluated

**Value**

A scalar value; the value of the Bartlett kernel function at the point value `x`.

**block_bootstrap**

Performs a block bootstrap on the functional data `f_data` with block size `b`.

**Description**

block_bootstrap Performs a block bootstrap on the functional data `f_data` with block size `b`.

**Usage**

`block_bootstrap(f_data, b, B = 300, moving = FALSE)`

**Arguments**

- `f_data`  
  the functional data matrix with observed functions in the columns
- `b`  
  the block size (of each block in each bootstrap sample)
- `B`  
  the number of bootstraps samples
- `moving`  
  boolean value specifying whether the block bootstrap should be moving or not.  
  A moving block bootstrap samples individual functional observations and adds on the consequent block, rather than sampling blocks of the data.

**Value**

Returns a list of `B` elements, each element being a block bootstrap sample in the same format as the original functional data `f_data`. 
brown_motion

Description

brown_motion Creates at J x N matrix, containing N independent Brownian motion sample paths in each of the columns.

Usage

brown_motion(N, J)

Arguments

N  the number of independent Brownian motion sample paths to compute.
J  the number of steps observed for each sample path (the resolution of the data).

Value

A J x N matrix containing Brownian motion functional data in the columns.

Examples

b <- brown_motion(250, 50)

B_h_bound

Description

B_h_bound Computes an approximate asymptotic upper 1-alpha confidence bound for the functional autocorrelation coefficient at lag h under a weak white noise assumption.

Usage

B_h_bound(f_data, lag, alpha = 0.05, M = NULL, low_disc = FALSE)
Arguments

- `f_data`: the functional data matrix with observed functions in the columns
- `lag`: the lag to use to compute the single lag test statistic
- `alpha`: the significance level to be used in the hypothesis test
- `M`: Number of samples to take when applying a Monte-Carlo approximation
- `low_disc`: Boolean value indicating whether or not to use low-discrepancy sampling in the Monte Carlo method. Note, low-discrepancy sampling will yield deterministic results.

Value

Numeric value; the 1-alpha confidence bound for the functional autocorrelation coefficient at lag h under a weak white noise assumption.

---

**B_iid_bound**

*Compute strong white noise confidence bound for autocorrelation coefficient.*

Description

`B_iid_bound` Computes an approximate asymptotic upper 1-alpha confidence bound for the functional autocorrelation coefficient at lag h under the assumption that `f_data` forms a strong white noise.

Usage

`B_iid_bound(f_data, alpha = 0.05)`

Arguments

- `f_data`: the functional data matrix with observed functions in the columns
- `alpha`: the significance level to be used in the hypothesis test

Value

Numeric value; the 1-alpha confidence bound for the functional autocorrelation coefficient at lag h under a strong white noise assumption.
**center**  
*Center functional data*

**Description**

center Centers the given functional data

**Usage**

```r
center(f_data)
```

**Arguments**

- `f_data`  
the functional data matrix with observed functions in the columns

**Value**

A matrix of the same form as `f_data` containing the centered functional data.

---

**covariance_diag_store**  
*List storage of diagonal covariances.*

**Description**

covariance_diag_store Creates a list storage of approximate diagonal covariances computed by the function `diagonal_covariance_i`

**Usage**

```r
covariance_diag_store(f_data, K)
```

**Arguments**

- `f_data`  
the functional data matrix with observed functions in the columns
- `K`  
the range of lags 1:K to use

**Value**

A list containing K 2-dimensional arrays containing the diagonal covariance matrices of the functional data, for lags h in the range 1:K.
covariance_i_j

<table>
<thead>
<tr>
<th>covariance_i_j</th>
<th>Compute the approximate covariance tensor for lag windows defined by i,j</th>
</tr>
</thead>
</table>

**Description**

covariance_i_j Computes the approximate covariance tensor of the functional data for lag windows defined by i,j.

**Usage**

covariance_i_j(f_data, i, j)

**Arguments**

- **f_data**: the functional data matrix with observed functions in the columns
- **i, j**: the indices i,j in 1:T that we are computing the covariance for

**Value**

A 4-dimensional array, encoding the covariance tensor of the functional data for lag windows defined by i,j.

covariance_i_j_vec

<table>
<thead>
<tr>
<th>covariance_i_j_vec</th>
<th>Compute the approximate covariance tensor for lag windows defined by i,j</th>
</tr>
</thead>
</table>

**Description**

covariance_i_j_vec Computes the approximate covariance tensor of the functional data for lag windows defined by i,j; a vectorized version of covariance_i_j.

**Usage**

covariance_i_j_vec(f_data, i, j)

**Arguments**

- **f_data**: the functional data matrix with observed functions in the columns
- **i, j**: the indices i,j in 1:T that we are computing the covariance for

**Value**

A 4-dimensional array, encoding the covariance tensor of the functional data for lag windows defined by i,j.
### daniell_kernel

**Daniell Kernel Function**

**Description**

*daniell_kernel* Computes the Daniell kernel function at a given point value.

**Usage**

```r
daniell_kernel(x)
```

**Arguments**

- `x` the point value at which the kernel function is evaluated

**Value**

A scalar value; the value of the Daniell kernel function at the point value `x`.

### diagonal_autocov_approx_0

**Compute the diagonal covariance**

**Description**

*diagonal_autocov_approx_0* Computes the diagonal covariance of the given functional data.

**Usage**

```r
diagonal_autocov_approx_0(f_data)
```

**Arguments**

- `f_data` the functional data matrix with observed functions in the columns

**Value**

A numeric value; integral approximation of the diagonal covariance of the functional data.
diagonal_covariance_i  

Compute the approximate diagonal covariance matrix for lag windows defined by i.

Description

diagonal_covariance_i Computes the approximate diagonal covariance matrix of the functional data for lag windows defined by i.

Usage

diagonal_covariance_i(f_data, i)

Arguments

f_data  the functional data matrix with observed functions in the columns
i  the index in 1:T that we are computing the covariance for

Value

A 2-dimensional array, encoding the covariance matrix of the functional data for lag windows defined by i.

far_1_S  

far_1_S Simulates an FAR(1,S)-fGARCH(1,1) process with N independent observations, each observed discretely at J points on the interval [0,1].

Description

far_1_S Simulates an FAR(1,S)-fGARCH(1,1) process with N independent observations, each observed discretely at J points on the interval [0,1].

Usage

far_1_S(N, J, S, type = "IID", burn_in = 50)

Arguments

N  the number of fGARCH(1,1) curves to sample.
J  the number of points at which each curve is sampled (the resolution of the data).
S  the autoregressive operator of the model, between 0 and 1, indicating the level of conditional heteroscedasticity.
the assumed model of the error term. The default argument is 'IID', under which the errors are assumed to be independent and identically distributed. The alternative argument is 'fGARCH', which will assume that the errors follow an fGARCH(1,1) process.

**burn_in**
the number of initial samples to burn (discard).

**Value**
A J x N matrix containing FAR(1,S) functional data in the columns.

**Examples**
```r
f <- far_1_S(100, 50, 0.75)
```

---

**fgarch_1_1**

Simulates an fGARCH(1,1) process with N independent observations, each observed

**Usage**
```r
fgarch_1_1(N, J, delta = 0.01, burn_in = 50)
```

**Arguments**
- **N**
  the number of fGARCH(1,1) curves to sample.
- **J**
  the number of points at which each curve is sampled (the resolution of the data).
- **delta**
  a parameter used in the variance recursion of the model.
- **burn_in**
  the number of initial samples to burn (discard).

**Value**
A list containing two J x N matrices, the former containing the sample of fGARCH(1,1) curves and the latter containing the respective variance values.

**Examples**
```r
f <- fgarch_1_1(100, 50)
```
Compute Functional Hypothesis Tests

**Description**

`fport_test` Computes a variety of functional portmanteau hypothesis tests. All hypothesis tests in this package are accessible through this function.

**Usage**

```r
fport_test(
  f_data,
  test = "multi-lag",
  lag = NULL,
  iid = FALSE,
  M = NULL,
  low_disc = FALSE,
  kernel = "Bartlett",
  bandwidth = "adaptive",
  components = 3,
  bootstrap = FALSE,
  block_size = "adaptive",
  moving = FALSE,
  straps = 300,
  alpha = 0.05,
  complete_test = FALSE,
  suppress_raw_output = FALSE,
  suppress_print_output = FALSE
)
```

**Arguments**

- **f_data**  
  The functional data matrix with observed functions in the columns.

- **test**  
  A String specifying the hypothesis test. Currently available tests are referred to by their string handles: "single-lag", "multi-lag", "spectral", "independence", and "imhof". Please see the Details section of the documentation, or the vignette, for a short overview of the available tests. For a more complete treatment of these hypothesis tests, please consult the references.

- **lag**  
  A positive integer value. Only used for the "single-lag", "multi-lag", "independence", and "imhof" tests. This parameter specifies the single lag, or maximum lag, to be used by the specified test.

- **iid**  
  Only used for the "single-lag" and "multi-lag" tests. A Boolean value, FALSE by default. If given TRUE, the hypothesis test will use a strong-white noise assumption (instead of a weak-white noise assumption).
M
Only used for the "single-lag" and "multi-lag" tests. A positive Integer. Determines the number of Monte-Carlo simulations employed in the Welch-Satterthwaite approximation of the limiting distribution of the test statistic.

low_disc
Only used for the "single-lag" and "multi-lag" tests. A Boolean value, FALSE by default. If given TRUE, uses low-discrepancy sampling in the Monte-Carlo method. Note, low-discrepancy sampling will yield deterministic results. Requires the 'fOptions' package.

kernel
Only used for the "spectral" test. A String, 'Bartlett' by default. Specifies the kernel to be used in the "spectral" test. Currently supported kernels are the 'Bartlett' and 'Parzen' kernels.

bandwidth
Only used for the "spectral" test. Either a String or a positive Integer value, 'adaptive' by default. Determines the bandwidth (or lag-window) to be used for the test. Given the string handle 'adaptive', the bandwidth is computed via a bandwidth selection method which aims to minimize the integrated normed error of the spectral density operator. If the given string handle is 'static', the bandwidth is computed to be \( n^{(1/(2q + 1))} \), where \( n \) is the sample size and \( q \) is the kernel order. If a positive integer is given, that will be the bandwidth that is used.

components
Only used for the "independence" test. A positive Integer value. Determines the number of functional principal components to use (ranked by their importance).

bootstrap
Only used for the "single-lag" test. A Boolean value, FALSE by default. If given TRUE, the hypothesis test is evaluated by approximating the limiting distribution of the test statistic via a block bootstrapping process.

block_size
Only used for the "single-lag" test in the case when 'bootstrap' = TRUE. A positive Integer value, with the default value being computed via the adaptive bandwidth selection method in the "spectral" test. Determines the block size (of each block in each bootstrap sample) if the test is being bootstrapped.

moving
Only used for the "single-lag" test in the case when 'bootstrap' = TRUE. A Boolean value, FALSE by default. If given TRUE, the performed block bootstrap will be moving rather than stationary.

straps
Only used for the "single-lag" test in the case when 'bootstrap' = TRUE. A positive Integer with a default value of 300. Determines the number of bootstrap samples to take if the test is being bootstrapped.

alpha
Numeric value between 0 and 1 specifying the significance level to be used in the specified hypothesis test. The default value is 0.05. Note, the significance value is only ever used to compute the 1-alpha quantile of the limiting distribution of the specified test's test statistic.

complete_test
A Boolean value, FALSE by default. If TRUE, the function requires no other parameters other than f_data, and will return a table with a single column containing p-values from an array of tests contained in the rows.

suppress_raw_output
A Boolean value, FALSE by default. If given TRUE, the function will not return a list containing the p-value, quantile and statistic, and instead only prints output to the console.

suppress_print_output
A Boolean value, FALSE by default. If TRUE, the function will not print any output to the console.
Details

The "single-lag" portmanteau test is based on the sample autocovariance function computed from the functional data. This test assesses the significance of lagged autocovariance operators at a single, user-specified lag h. More specifically, it tests the null hypothesis that the lag-h autocovariance operator is equal to 0. This test is designed for stationary functional time-series, and is valid under conditional heteroscedasticity conditions. The required parameter for this test is 'lag', which determines the lag at which the test is evaluated. If this parameter is left blank, it will take a default of 1. The optional parameters for this test are 'iid', 'M', 'low_disc', 'bootstrap', 'block_size', 'straps', 'moving', and 'alpha'.

The "multi-lag" portmanteau test is also based on the sample autocovariance function computed from the functional data. This test assesses the cumulative significance of lagged autocovariance operators, up to a user-selected maximum lag K. More specifically, it tests the null hypothesis that the first K lag-h autocovariance operators (h going from 1 to K) is equal to 0. This test is designed for stationary functional time-series, and is valid under conditional heteroscedasticity conditions. The required parameter for this test is 'lag', which determines the maximum lag at which the test is evaluated. If this parameter is left blank, it will take a default of 20. The optional parameters for this test are 'iid', 'M', 'low_disc', 'bootstrap', 'block_size', 'straps', 'moving', and 'alpha'.

The "spectral" portmanteau test is based on the spectral density operator. It essentially measures the proximity of a functional time series to a white noise - the constant spectral density operator of an uncorrelated series. Unlike the "single-lag" and "multi-lag" tests, this test is not for general white noise series, and may not hold under functional conditionally heteroscedastic assumptions. The optional parameters for this test are 'kernel', 'bandwidth', and 'alpha'.

The "independence" portmanteau test is a test of independence and identical distribution based on a dimensionality reduction by projecting the data onto the most important functional principal components. It is based on the resulting lagged cross-variances. This test is not for general white noise series, and may not hold under functional conditionally heteroscedastic assumptions. The required parameters for this test are 'lag' and 'components'. The 'lag' parameter determines the maximum lag at which the test is evaluated. The 'components' parameter determines the number of the most important principal components to use (importance is determined by the proportion of the variance that is explained by the individual principal component.)

The "imhof" portmanteau test is an analogue of the "single-lag" test. While the "single-lag" test computes the limiting distribution of the test statistic via a Welch-Satterthwaite approximation, the "imhof" test directly computes the coefficients of the quadratic form in Normal variables which the test statistic converges too as the sample size goes to infinity. We warn the user that this test is extremely computationally expensive, and is only recommended for small datasets as a means of cross-verification against the single-lag test. The required parameter for this test is 'lag', which determines the lag at which the test is evaluated. The "imhof" test requires the "tensorA" and "CompQuadForm" packages. Note also that the imhof test does not return a statistic, and thus returns a list with only 2 elements if suppress_raw_output = FALSE.

Value

If suppress_raw_output = FALSE, a list containing the test statistic, the 1-alpha quantile of the limiting distribution, and the p-value computed from the specified hypothesis test. Also prints output containing a short description of the test, the p-value, and additional information about the test if suppress_print_output = FALSE. If 'complete-test' = TRUE, will return a 1-column table
instead containing the p-values for a variety of tests, which are given short descriptions in the index of the table.

References


Examples

```R
b <- brown_motion(250, 50)
fport_test(b, test = 'single-lag', lag = 10)
fport_test(b, test = 'multi-lag', lag = 10, alpha = 0.01)
fport_test(b, test = 'single-lag', lag = 1, M = 250, low_disc = TRUE)
fport_test(b, test = 'spectral', kernel = 'Bartlett', bandwidth = 'static', alpha = 0.05)
fport_test(b, test = 'spectral', alpha = 0.1, kernel = 'Parzen', bandwidth = 'adaptive')
fport_test(b, test = 'independence', components = 3, lag = 3)
```

### iid_covariance

** iid_covariance **

*Compute part of the covariance under a strong white noise assumption*

#### Description

** iid_covariance ** A helper function used to compute one of the two independent sum terms in the computation of the approximate covariance of the functional data under a strong white noise assumption.

#### Usage

```R
iid_covariance(f_data)
```

#### Arguments

- **f_data** the functional data matrix with observed functions in the columns

#### Value

A 2-dimensional matrix containing one of the two independent sums in the computation of the covariance.
**iid_covariance_vec**  
*Compute part of the covariance under a strong white noise assumption*

### Description

`iid_covariance_vec` A helper function used to compute one of the two independent sum terms in the computation of the approximate covariance of the functional data under a strong white noise assumption; a vectorized version of `iid_covariance`.

### Usage

```r
iid_covariance_vec(f_data)
```

### Arguments

- **f_data**  
  the functional data matrix with observed functions in the columns

### Value

A 2-dimensional matrix containing one of the two independent sums in the computation of the covariance.

---

**independence_test**  
*Independence Test*

### Description

`independence_test` Computes the independence test with a user-specified number of principal components and range of lags.

### Usage

```r
independence_test(
  f_data,
  components,
  lag,
  alpha = 0.05,
  suppress_raw_output = FALSE,
  suppress_print_output = FALSE
)
```
Arguments

f_data the functional data matrix with observed functions in the columns

components A positive Integer specifying the number of principal components to project the data on; ranked in order of importance (importance is determined by the proportion of the variance that is explained by the individual principal component.)

lag A positive Integer value, specifying the maximum lag to include - this can be seen as the bandwidth or lag-window.

alpha Numeric value between 0 and 1 specifying the significance level to be used in the specified hypothesis test. The default value is 0.05. Note, the significance value is only ever used to compute the 1-alpha quantile of the limiting distribution of the specified test's test statistic.

suppress_raw_output

Boolean value, FALSE by default. If TRUE, the function will not return the list containing the p-value, quantile, and statistic.

suppress_print_output

Boolean value, FALSE by default. If TRUE, the function will not print any output to the console.

Details

The "independence" portmanteau test is a test of independence and identical distribution based on a dimensionality reduction by projecting the data onto the most important functional principal components. It is based on the resulting lagged cross-variances. This test is not for general white noise series, and may not hold under functional conditionally heteroscedastic assumptions. Please consult the vignette for a deeper exposition, and consult the reference for a complete treatment.

Value

If suppress_raw_output = FALSE, a list containing the test statistic, the 1-alpha quantile of the limiting distribution, and the p-value computed from the specified hypothesis test. Also prints output containing a short description of the test, the p-value, and additional information about the test if suppress_print_output = FALSE.

References


Examples

b <- brown_motion(250, 100)
independence_test(b, components = 3, lag = 5)
multi_lag_test

Description

multi_lag_test Computes the multi-lag hypothesis test over a range of user-specified lags.

Usage

multi_lag_test(
  f_data,
  lag = 20,
  M = NULL,
  low_disc = FALSE,
  iid = FALSE,
  alpha = 0.05,
  suppress_raw_output = FALSE,
  suppress_print_output = FALSE
)

Arguments

- **f_data**: the functional data matrix with observed functions in the columns
- **lag**: Positive integer value. The lag to use to compute the single lag test statistic
- **M**: Positive integer value. Number of Monte-Carlo simulation for Welch-Satterthwaite approximation.
- **low_disc**: A Boolean value, FALSE by default. If given TRUE, uses low-discrepancy sampling in the Monte-Carlo method. Note, low-discrepancy sampling will yield deterministic results. Requires the 'fOptions' package.
- **iid**: A Boolean value, FALSE by default. If given TRUE, the hypothesis test will use a strong-white noise assumption (instead of a weak-white noise assumption).
- **alpha**: Numeric value between 0 and 1 specifying the significance level to be used in the specified hypothesis test. The default value is 0.05. Note, the significance value is only ever used to compute the 1-alpha quantile of the limiting distribution of the specified test’s test statistic.
- **suppress_raw_output**: Boolean value, FALSE by default. If TRUE, the function will not return the list containing the p-value, quantile, and statistic.
- **suppress_print_output**: Boolean value, FALSE by default. If TRUE, the function will not print any output to the console.
Details

The "multi-lag" portmanteau test is also based on the sample autocovariance function computed from the functional data. This test assesses the cumulative significance of lagged autocovariance operators, up to a user-selected maximum lag K. More specifically, it tests the null hypothesis that the first K lag-h autocovariance operators (h going from 1 to K) is equal to 0. This test is designed for stationary functional time-series, and is valid under conditional heteroscedasticity conditions.

Value

If suppress_raw_output = FALSE, a list containing the test statistic, the 1-alpha quantile of the limiting distribution, and the p-value computed from the specified hypothesis test. Also prints output containing a short description of the test, the p-value, and additional information about the test if suppress_print_output = FALSE.

References


Examples

b <- brown_motion(150, 50)
multi_lag_test(b, lag = 5)
multi_lag_test(b, lag = 10, M = 50)

parzen_kernel

Description

parzen_kernel Computes the Parzen kernel function at a given point value.

Usage

parzen_kernel(x)

Arguments

x the point value at which the kernel function is evaluated

Value

A scalar value; the value of the Parzen kernel function at the point value x.
Q_WS_hyp_test

Compute size alpha single-lag hypothesis test under weak or strong white noise assumption

Description

Q_WS_hyp_test Computes the size alpha test of a single lag hypothesis under a weak white noise or strong white noise assumption using a Welch-Satterthwaite Approximation.

Usage

Q_WS_hyp_test(f_data, lag, alpha = 0.05, iid = FALSE, M = NULL, low_disc = FALSE, bootstrap = FALSE, block_size = "adaptive", straps = 300, moving = FALSE)

Arguments

- **f_data**: the functional data matrix with observed functions in the columns
- **lag**: the lag to use to compute the single lag test statistic
- **alpha**: the significance level to be used in the hypothesis test
- **iid**: boolean value, if given TRUE, the hypothesis test will use a strong-white noise assumption. By default is FALSE, in which the hypothesis test will use a weak-white noise assumption.
- **M**: Number of samples to take when applying a Monte-Carlo approximation
- **low_disc**: Boolean value indicating whether or not to use low-discrepancy sampling in the Monte Carlo method. Note, low-discrepancy sampling will yield deterministic results.
- **bootstrap**: boolean value, if given TRUE, the hypothesis test is done by approximating the limiting distribution of the test statistic via a block bootstrap algorithm. FALSE by default
- **block_size**: the block size to be used in the block bootstrap method (in each bootstrap sample). 10 by default.
- **straps**: the number of bootstrap samples to take; 300 by default
- **moving**: boolean value; determines whether or not the block bootstrap should be moving
scalar_covariance_i_j Computes the approximate covariance at a point for lag windows defined by i,j

Description
scalar_covariance_i_j Computes the approximate covariance at a point of the functional data for lag windows defined by i,j; a scalarized version of covariance_i_j that takes point estimates.

Usage
scalar_covariance_i_j(f_data, i, j, times)

Arguments
f_data the functional data matrix with observed functions in the columns
i, j the indices i,j in 1:T that we are computing the covariance for
times A vector with 4 columns containing indices specifying which subset of f_data to consider

Value
A numeric value; the covariance of the functional data at a point for lag windows defined by i,j.

scalar_covariance_i_j_vec Computes the approximate covariance at a point for lag windows defined by i,j

Description
scalar_covariance_i_j_vec Computes the approximate covariance at a point of the functional data for lag windows defined by i,j; a vectorized version of scalar_covariance_i_j.

Usage
scalar_covariance_i_j_vec(f_data, i, j, times)

Value
A list containing the p-value, the quantile, and a boolean value indicating whether or not the hypothesis is rejected.
Arguments

- **f_data**: the functional data matrix with observed functions in the columns.
- **i, j**: the indices i,j in 1:T that we are computing the covariance for.
- **times**: A vector with 4 columns containing indices specifying which subset of f_data to consider.

Value

A numeric value; the covariance of the functional data at a point for lag windows defined by i,j.

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**single_lag_test**

**Single-Lag Hypothesis Test**

Description

single_lag_test Computes the single-lag hypothesis test at a single user-specified lag.

Usage

```r
single_lag_test(
  f_data,
  lag = 1,
  alpha = 0.05,
  iid = FALSE,
  M = NULL,
  low_disc = FALSE,
  bootstrap = FALSE,
  block_size = "adaptive",
  straps = 300,
  moving = FALSE,
  suppress_raw_output = FALSE,
  suppress_print_output = FALSE
)
```

Arguments

- **f_data**: The functional data matrix with observed functions in the columns.
- **lag**: Positive integer value. The lag to use to compute the single lag test statistic.
- **alpha**: Numeric value between 0 and 1 specifying the significance level to be used in the specified hypothesis test. The default value is 0.05. Note, the significance value is only ever used to compute the 1-alpha quantile of the limiting distribution of the specified test’s test statistic.
- **iid**: A Boolean value, FALSE by default. If given TRUE, the hypothesis test will use a strong-white noise assumption (instead of a weak-white noise assumption).
\texttt{M} Positive integer value. Number of Monte-Carlo simulations for the Welch-Satterthwaite approximation.

\texttt{low\_disc} A Boolean value, FALSE by default. If given TRUE, uses low-discrepancy sampling in the Monte-Carlo method. Note, low-discrepancy sampling will yield deterministic results. Requires the 'fOptions' package.

\texttt{bootstrap} A Boolean value, FALSE by default If given TRUE, the hypothesis test is done by approximating the limiting distribution of the test statistic via a block bootstrap process.

\texttt{block\_size} A positive Integer value, with the default value being computed via the adaptive bandwidth selection method in the "spectral" test. Determines the block size (of each block in each bootstrap sample) if the test is being bootstrapped.

\texttt{straps} A positive Integer, with a default value of 300. Determines the number of bootstrap samples to take if the test is being bootstrapped. Only used if \texttt{bootstrap} == TRUE.

\texttt{moving} A Boolean value, FALSE by default If given TRUE, the performed block bootstrap will be moving rather than stationary.

\texttt{suppress\_raw\_output} Boolean value, FALSE by default. If TRUE, the function will not return the list containing the p-value, quantile, and statistic.

\texttt{suppress\_print\_output} Boolean value, FALSE by default. If TRUE, the function will not print any output to the console.

**Details**

The "single-lag" portmanteau test is based on the sample autocovariance function computed from the functional data. This test assesses the significance of lagged autocovariance operators at a single, user-specified lag h. More specifically, it tests the null hypothesis that the lag-h autocovariance operator is equal to 0. This test is designed for stationary functional time-series, and is valid under conditional heteroscedasticity conditions.

**Value**

If \texttt{suppress\_raw\_output} = FALSE, a list containing the test statistic, the 1-alpha quantile of the limiting distribution, and the p-value computed from the specified hypothesis test. Also prints output containing a short description of the test, the p-value, and additional information about the test if \texttt{suppress\_print\_output} = FALSE.

**References**


**Examples**

```r
f <- far_1_S(150, 50, S = 0.75)
single_lag_test(f, lag = 1)
single_lag_test(f, lag = 2, M=100)
```
**spectral_test**

**Spectral Density Test**

**Description**

The "spectral" portmanteau test is based on the spectral density operator. It essentially measures the proximity of a functional time series to a white noise - the constant spectral density operator of an uncorrelated series. Unlike the "single-lag" and "multi-lag" tests, this test is not for general white noise series, and may not hold under functional conditionally heteroscedastic assumptions.

**Usage**

```r
spectral_test(
  f_data,
  kernel = "Bartlett",
  bandwidth = "adaptive",
  alpha = 0.05,
  suppress_raw_output = FALSE,
  suppress_print_output = FALSE
)
```

**Arguments**

- **f_data**: The functional data matrix with observed functions in the columns.
- **kernel**: A String specifying the kernel function to use. The currently supported kernels are the 'Bartlett' and 'Parzen' kernels. The default kernel is 'Bartlett'.
- **bandwidth**: A String or positive Integer value which specifies the bandwidth to use. Currently admitted string handles are 'static' which computes the bandwidth $p$ via $p = n^{\gamma(1/(2q+1))}$ where $n$ is the sample size and $q$ is the kernel order, or 'adaptive' which uses a bandwidth selection method that is based on the functional data.
- **alpha**: Numeric value between 0 and 1 specifying the significance level to be used for the test. The significance level is 0.05 by default. Note, the significance value is only ever used to compute the 1-alpha quantile of the limiting distribution of the specified test’s test statistic.
- **suppress_raw_output**: Boolean value, FALSE by default. If TRUE, the function will not return the list containing the p-value, quantile, and statistic.
- **suppress_print_output**: Boolean value, FALSE by default. If TRUE, the function will not print any output to the console.

**Details**

**spectral_test** Computes the spectral hypothesis test under a user-specified kernel function and bandwidth; automatic bandwidth selection methods are provided.
Value

If suppress_raw_output = FALSE, a list containing the test statistic, the 1-alpha quantile of the limiting distribution, and the p-value computed from the specified hypothesis test. Also prints output containing a short description of the test, the p-value, and additional information about the test if suppress_print_output = FALSE.

References


Examples

```r
b <- brown_motion(100, 50)
spectral_test(b)
spectral_test(b, kernel = 'Parzen', bandwidth = 'adaptive')
spectral_test(b, kernel = 'Bartlett', bandwidth = 2)
```
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